ONLINE APPENDIX: Vertical Integration and Exclusivity in Platform and Two-Sided Markets

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A Additional Details

A.1 Consumer Demand

A.1.1 Estimation and Computation

Recall that $\iota \in \mathcal{I} \equiv \{0,1\}^3$ denotes a consumer's inventory state. Slightly abusing notation, also let $\iota = \mathbf{1}_{PS2} + \mathbf{1}_{XB} \times 2 + \mathbf{1}_{GC} \times 4$, where $\mathbf{1}_j$ is an indicator for console j being owned. I discretize the distributions of α^p and α^γ to model consumer heterogeneity: consumers are divided among R groups indexed by i, each with price-sensitivity and gaming-preference coefficients $(\alpha_i^p, \alpha_i^\gamma)$ and initial population shares $\lambda_{i,t=0,\iota=0}$ obtained via independent univariate Gauss-Hermite quadrature (c.f. Judd (1998); Heiss and Winschel (2007)). In estimation, α^γ was allowed to take on 11 distinct values and α^p 5 values, resulting in $R = 55$ distinct consumer types. However, due to the difficulty in identifying heterogeneity in α^p, only heterogeneity in α^γ was ultimately allowed.

Let $\lambda_{i,t,\iota}$ represent the share of the population comprising a consumer of type i with inventory ι at time t. An overview of the estimation routine is:

- For a candidate θ, iterate on the following until convergence is obtained on $\{\Gamma_{j,t}(\alpha_i^\gamma, \alpha_i^{p,hw}; \iota), \lambda_{i,t,\iota}\}_{\forall t,i,\iota}$ (where a tolerance of 10^{-6} in the sup-norm was used for Γ):

  i Hardware Adoption: At iteration n, for a given $\{\Gamma^n_{j,t}(\alpha_i^\gamma, \alpha_i^{p,hw}; \iota)\}_{\forall j \in \mathcal{J}_t, t, i, \iota}$, determine mean console utilities $\{\delta^{n+1}_{i,j,t,\iota}\}_{\forall i, j \in \mathcal{J}_t, t, \iota}$ which match observed shares in data with those predicted by the model. Update the distribution of consumer types with each inventory $\{\lambda^{n+1}_{i,t,\iota}\}_{\forall t,i,\iota}$.

  ii Software Adoption:
  Given the distribution of consumers onboard any hardware platform, compute mean software utilities $\{\zeta_{j,k,t}\}_{j \in \mathcal{J}_t, k \in \mathcal{K}_{j,t}, t}$ for every software title on every platform that, again, match observed shares in data with those predicted by the model. Update implied software utilities $\{\Gamma^{n+1}_{j,t}(\alpha_i^\gamma, \alpha_i^{p,hw}; \iota)\}_{\forall j \in \mathcal{J}_t, t, i, \iota}$.

- Form innovations in product unobservables:

$$\{\nu^{hw}_{j,t}\}_{j \in \mathcal{J}_t, \forall t} = \{\xi_{j,t}(\theta) - \rho^{hw} \xi_{j,t-1}(\theta)\}_{j \in \mathcal{J}_t, \forall t}$$
$$\{\nu^{sw}_{j,k,t}\}_{j \in \mathcal{J}_t, k \in \mathcal{K}_{j,t}, \forall t} = \{\eta_{j,k,t}(\theta) - \rho^{sw} \eta_{j,k,t-1}(\theta)\}_{j \in \mathcal{J}_t, k \in \mathcal{K}_{j,t}, \forall t}$$

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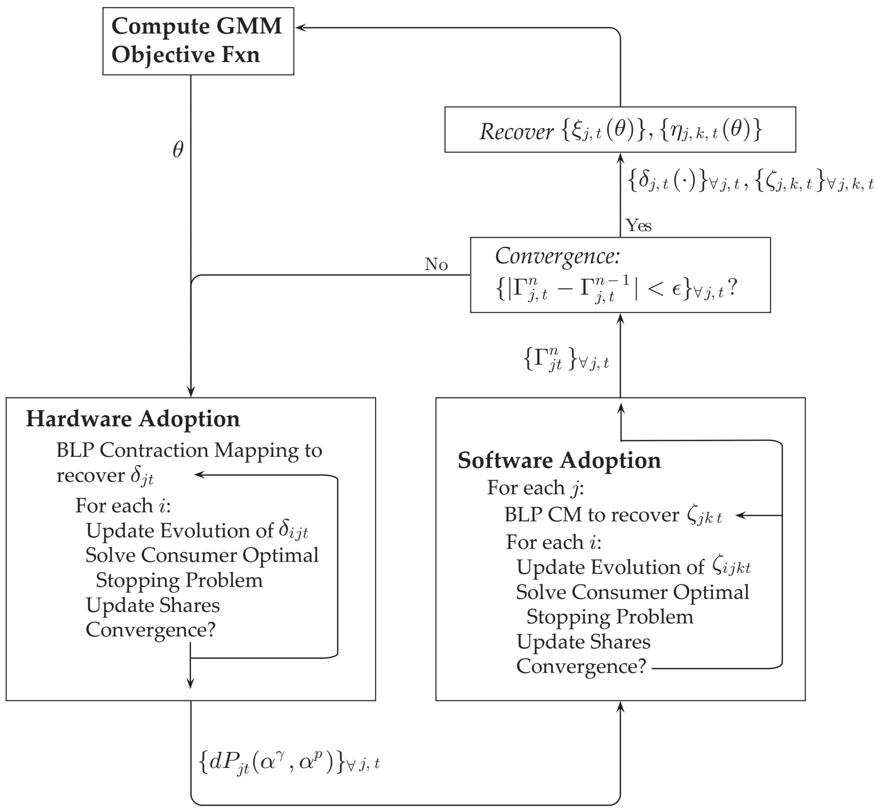
Figure 1: Estimation Algorithm



Notes: Computation algorithm for estimation of consumer demand. "BLP Contraction Mapping" and "BLP CM" refers to the contraction mapping given by (3) introduced in Berry, Levinsohn and Pakes (1995).

and compute the GMM objective.

The algorithm is summarized Figure 1. Additional details follow.

Hardware Adoption Note that the values $\{\delta_{i,j,t,\iota}\}_{j\in\mathcal{J}_t,\iota\in\mathcal{I}}$ are sufficient to compute the expected probabilities that a consumer with any inventory ι, prior to realizing $\epsilon_{i,t}$, will purchase any hardware console at time t:

$$\hat{s}_{i,t,\iota}(\{\delta_{i,j,t,\iota}\}_{j\in\mathcal{J}_t}) = \frac{\exp(\delta_{i,t,\iota})}{\exp(\delta_{i,t,\iota}) + \exp(\beta E[EV_i(\{\delta_{i,j,t+1,\iota}\}_{j\in\mathcal{J}_{t+1}}, \iota, m(t+1))|\{\delta_{i,j,t,\iota}\}_{j\in\mathcal{J}_t}, \iota, m(t)])} \tag{1}$$

as well as the probability a consumer purchases a particular hardware platform j conditional on purchasing any platform:

$$\hat{s}_{i,j,\iota|t}(\{\delta_{i,j,t,\iota}\}_{j\in\mathcal{J}_t}) = \frac{\exp(\delta_{i,j,t,\iota} + \beta E[EV_i(\{\delta_{i,j,t+1,\iota}\}_{j\in\mathcal{J}_{t+1},\iota\in\mathcal{I}}, \iota\cup\{j\}, m(t+1)|\{\delta_{i,j,t,\iota}\}_{j\in\mathcal{J}_t,\iota\in\mathcal{I}})]))}{\exp(\delta_{i,t,\iota})} \tag{2}$$

where $\delta_{i,t,\iota} = \ln(\sum_{j\notin\iota}\exp(\delta_{i,j,t,\iota} + \beta E[EV_i(\{\delta_{i,j,t+1,\iota}\}_{j\in\mathcal{J}_{t+1},\iota\in\mathcal{I}}, \iota\cup\{j'\}, m(t+1))|\{\delta_{i,j,t,\iota}\}_{j\in\mathcal{J}_t,\iota\in\mathcal{I}}]))$.[1] Thus, provided the values of $\{\delta_{i,j,t,\iota}\}_{j\in\mathcal{J}_t,\iota\in\mathcal{I}}$ for each consumer i, aggregation over (1) and (2) yields

[1] These are the standard "logit" closed form expressions obtained from integrating over $\epsilon_{i,t}$.

the total predicted share of consumers that purchases console j at time t:

$$\hat{s}_{j,t}(\{\delta_{i,j,t,\iota}\}_{\forall i,j\in\mathcal{J}_t,\forall\iota},\{\lambda_{i,t,\iota}\})=\sum_{\iota,i}\hat{s}_{i,t,\iota}(\cdot)\hat{s}_{i,j,\iota|t}(\cdot)\lambda_{i,t,\iota}$$

where this term can be matched to $s^o_{j,t}$, which represents the share of potential buyers who purchase console j at time t observed the data. The distribution of consumers across platforms given by $\lambda_{i,t,\iota}$ changes over time according to the population of consumers who have purchased in previous periods; this is one of the primary ways the demand system generates interdependence over time.

Define the mean utility for the "mean" consumer ($i = 0$) of hardware platform j at time t and inventory state $\iota = 0$ as

$$\delta_{j,t}\equiv\alpha^x\mathbf{x}_{j,t}+\alpha_0^{p,hw}p_{j,t}+\alpha^\Gamma\Gamma_{j,t}(\alpha_0^\gamma,\alpha_0^{p,hw};\iota=0)+\xi_{j,t}$$

For a fixed θ_1, $\{\Gamma_{j,t}(\alpha_i^\gamma,\alpha_i^{p,hw};\iota)\}_{\forall j\in\mathcal{J}_t,t,i,\iota}$, and $\{\lambda^j_{i,0,\iota}\}_{\forall i,j\in\mathcal{J}_t,\iota}$, the contraction mapping introduced in Berry, Levinsohn and Pakes (1995):

$$\delta^n_{j,t}=\delta^{n-1}_{j,t}+\psi\left(\ln(s^o_{j,t})-\ln(\hat{s}_{j,t}(\cdot))\right)\tag{3}$$

is used to obtain to mean console qualities $\delta_{j,t}(\cdot)$, where $\psi\in(0,1)$. Using the sup-norm, the tolerance for δ was set to 10^{-10}.

- At iteration m of the mapping, implied market shares $\hat{s}_{j,t}(\cdot)$ are updated:

$$\hat{s}^m_{j,t}(\{\delta^{m-1}_{j,t}\},\{\lambda_{i,t,\iota}\})=\sum_{\iota=0}^{6}\sum_{i=1}^{R}\lambda_{i,t,\iota}\hat{s}_{i,t,\iota}(\cdot)\hat{s}_{i,j,\iota|t}(\cdot)$$

  - To obtain $\hat{s}_{i,t,\iota}(\cdot)$ and $\hat{s}_{i,j,\iota|t}(\cdot)$, the consumer dynamic optimization problem for a given $\delta^{m-1}_{j,t}$ and for every consumer type i and inventory state ι must be solved. Noting $\delta_{i,j,t,\iota}=\delta_{j,t}-(\alpha_i^p-\alpha_0^p)p_{j,t}+\alpha^\Gamma(\Gamma_{j,t}(\alpha_i^\gamma,\alpha_i^p;\iota)-\Gamma_{j,t}(\alpha_0^\gamma,\alpha_0^p;\iota=0))$, beliefs $F_{i,\iota}(\cdot)$ are updated according to the regression given by (8) in the main text. I assume that there is a finite horizon $\bar{T}$ at which point $\{\delta_{j,\bar{T}(\iota)}\}_{\forall j\in\mathcal{J}_t,\iota}$ decays to 0, and simulate forward 10 sample paths to compute the expected value function $\{EV_i(\{\delta_{i,j,t,\iota}\},\iota,m(t))\}_{\forall i,\iota}$.[2] In practice, I assume that this horizon occurs in January 2006, 3 months after the end of the data sample; however, results did not change substantially when the horizon was extended by 6 months or an additional year.
  - To update $\{\lambda_{i,t,\iota}\}_{\forall i,t>0,\iota}$, first shares $\{\lambda_{i,t=0,\iota=0}\}_{\forall i}$ are computed from the distribution implied by θ_1, and then each future period is computed by updating the distribution of consumers remaining on the market as follows:

$$\lambda_{i,t+1,\iota}=\frac{(1-\hat{s}_{i,t,\iota})\lambda_{i,t,\iota}+\sum_{\iota'\in\mathbf{I}^-(\iota)}\lambda_{i,t,\iota'}\hat{s}_{i,t,\iota'}\hat{s}_{i,\{\iota\setminus\iota'\},\iota'|t}}{\sum_{i,\iota''\neq 7}\lambda_{i,t+1,\iota''}}\tag{4}$$

    where $\mathbf{I}^-(\iota)$ is the set of inventory states that can "reach" inventory state ι—e.g., differ only by having one fewer console. In other words, the share of consumers with inventory ι at time $t+1$ are simply those that did not purchase a new console at time t (the first term of the numerator) plus those in state ι' at time t who purchase console j, where j is

[2] I also explored using a discretized state space with a non-uniform grid (concentrating points in areas that are more likely to be visited), simplical interpolation, and standard value function iteration for convergence.

the only difference between ι and ι'. To account for the growth in total market size (i.e., more television households are present in each period), I assume that new households are distributed across consumer types according to their initial distribution.[3]

– This inner loop—updating beliefs $F_{i,\iota}$ and EV_i for all consumers—is repeated until convergence.

Software Adoption First, note the distribution of consumers of type i onboard console j at time t can be obtained from $\{\lambda_{i,t,\iota}\}$: $\lambda^j_{i,t} = (\sum_{\iota|j\in\iota}\lambda_{i,t,\iota})/(\sum_i\sum_{\iota|j\in\iota}\lambda_{i,t,\iota})$. This will be a function of the hardware adoption decision for all consumers in periods $\tau \leq t$, and it is the evolution of this distribution over time and across platforms that necessitates the joint estimation of software and hardware demand.

As on the hardware side, note the mean utility for consumer i, $\zeta_{i,j,k,t}$, is a sufficient statistic in determining whether or not consumer i with platform j purchases software title k in a given period t. The share of consumers of type i who have not yet purchased title k, but purchase it in period t, is:

$$\hat{s}_{i,j,k,t}(\zeta_{i,j,k,t}) = \frac{\exp(\zeta_{i,j,k,t})}{\exp(\zeta_{i,j,k,t}) + \exp(\beta E[EW_i(\zeta_{i,j,k,t+1}|\zeta_{i,j,k,t}, m(t))])}\ , \tag{5}$$

and the share of all consumers who purchase title k is $\hat{s}_{j,k,t}(\{\zeta_{i,j,k,t}\}\forall i) = \sum_i \hat{s}_{i,j,k,t}\lambda^j_{i,k,t}$. where $\lambda^j_{i,k,t}$ is the share of consumers of type i onboard platform j who have not yet purchased software k at time t, and $\lambda^j_{i,k,t=r_k} = \lambda^j_{i,t=r_k}$, where r_k is title k's release date. This value will be matched to $s^o_{j,k,t}$, which is the observed share of consumers in the data who have purchased title k onboard platform j at time t.

To obtain a starting value of $\{\Gamma_{j,t}(\cdot;\iota)\}_{\forall\iota}$ for the hardware adoption side, I first assume that $\{\lambda^j_{i,t}\}\forall t = \lambda_{i,0,\iota=0}$—i.e., the entry distribution of consumer types on each hardware platform is equal to the initial distribution and stationary across time. For a given θ_1, $\{\lambda^j_{i,t}\}\forall i, j\in\mathcal{J}_t, t$, the software side proceeds in a parallel fashion to the hardware adoption side. For each console j, the same BLP contraction mapping is used to recover mean software qualities $\zeta_{j,k,t}$:

$$\zeta^m_{j,k,t}(\theta_1, \{\lambda^j_{i,t}\}) = \zeta^{m-1}_{j,k,t} + \ln(s^o_{j,k,t}) - \ln(\hat{s}_{j,k,t}(\zeta^{m-1}_{j,k,t}))$$

- Implied market shares $\hat{s}_{j,k,t}(\cdot)$ are computed as in the hardware side (except now there are only two inventory states {0,1}), where the initial base of consumers who have not purchased a title is given by the distribution of consumers on a given console at the time of the title's release, and each future period's potential market size is updated accordingly. Again, the consumer dynamic optimization problem for a given $\zeta^{m-1}_{j,k,t}$ is solved for every consumer type i, where $\zeta_{i,j,k,t} = \zeta_{j,k,t} - (\alpha^p_i - \alpha^p_0)p_{k,t} + \alpha^\gamma_i$. I discretize the state space into a uniform grid with 201×12 points, and employ Halton sequences for random draws on the evolution of $\zeta_{i,j,k,t}$, simple linear interpolation, and standard value function iteration for convergence. At each stage, beliefs are updated according to the regression given by (8) in the main text. The process repeats until convergence on beliefs and each title's expected value function is obtained. Using the sup-norm, tolerances were set to 10^{-10} for ζ and 10^{-14} for the expected value function EW.

Once the expected value function is computed for each software title, consumer type, and time period, Γ is updated.

[3]Since the # of television households grows by only 6M households (6%) during this time period, changing this assumption did not affect results significantly.

Recovery of $\nu^{hw}(\theta)$ and $\nu^{sw}(\theta)$ The hardware and software adoption algorithms are repeated until convergence on $\{\Gamma_{j,t}(\alpha_i^{\gamma}, \alpha_i^{p,hw}; \iota)\}_{j\in\mathcal{J}_t;\forall t,i,\iota}$ and $\{\lambda_{i,j,t,\iota}\}_{\forall i,j\in\mathcal{J}_t,t,\iota}$ is obtained. This yields $\{\delta_{j,t,0}(\theta)\}_{j\in\mathcal{J}_t;\forall t}$ and $\{\zeta_{j,k,t}(\theta)\}_{j\in\mathcal{J}_t,k\in\mathcal{K}_{j,t},t}$.

Note that θ_2 can be expressed as a function of θ_1 by using the first order conditions of the objective function:

$$\hat{\theta_2}(\theta_1) = (X'\mathbf{Z}\Phi^{-1}\mathbf{Z}'X)^{-1}X'\mathbf{Z}\Phi^{-1}\mathbf{Z}'Y$$

where

$$X = \begin{bmatrix} \Delta^{\rho^{hw}} x & 0 \\ \Delta^{\rho^{hw}}(\Delta^{\rho^{hw}} x) & 0 \\ 0 & \Delta^{\rho^{sw}} w \\ 0 & \Delta^{\rho^{sw}}(\Delta^{\rho^{sw}} w) \end{bmatrix}, \qquad \mathbf{Z} = \begin{bmatrix} \mathbf{Z}^{hw} & 0 \\ \mathbf{Z}^{hw,\Delta} & 0 \\ 0 & \mathbf{Z}^{sw} \\ 0 & \mathbf{Z}^{sw,\Delta} \end{bmatrix}, \qquad Y = \begin{bmatrix} \Delta^{\rho^{hw}} \tilde{\delta} \\ \Delta^{\rho^{hw}}(\Delta^{\rho^{hw}} \tilde{\delta}) \\ \Delta^{\rho^{sw}} \tilde{\zeta} \\ \Delta^{\rho^{sw}}(\Delta^{\rho^{sw}} \tilde{\zeta}) \end{bmatrix},$$

$\Delta^{\rho} x \equiv x_t - \rho x_{t-1}$, x and w are stacked characteristics across all hardware and software products over time, $\mathbf{Z}$ is the set of stacked instruments discussed in Section 4, Φ is the GMM weighting matrix $(\mathbf{Z}'\mathbf{Z})$, and $\tilde{\delta}$ and $\tilde{\zeta}$ is a vector of stacked adjusted product lifetime utilities, where $\tilde{\delta}_{j,t} = \delta_{j,t} + \alpha_0^{p,hw} p_{j,t} - \alpha^{\Gamma}\Gamma_{j,t}$ and $\tilde{\zeta}_{j,k,t} = \zeta_{j,k,t} + \alpha_0^{p,sw} p_{j,k,t}$. Since $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ are solely functions of θ_1 and the data, a non-linear search needs only be conducted over θ_1.

Finally, innovations in product unobservables which are used to form the GMM objective are computed as follows:

$$\begin{aligned} \nu_{j,t}^{hw} &= (\delta_{j,t} - \rho^{hw}\delta_{j,t-1}) - \alpha_0^{p,hw}(p_{j,t} - \rho^{hw} p_{j,t-1}) \\ &\qquad - \alpha^{\Gamma}(\Gamma_{j,t}(\cdot;\iota=0) - \rho^{hw}\Gamma_{j,t-1}(\cdot;\iota=0)) - \alpha^{x}(\mathbf{x}_{j,t} - \rho^{hw}\mathbf{x}_{j,t-1}) \\ \nu_{j,k,t}^{sw} &= (\zeta_{j,k,t} - \rho^{sw}\zeta_{j,k,t}) - \alpha_0^{p,sw}(p_{j,k,t} - \rho^{sw} p_{j,k,t-1}) - \alpha^{w}(\mathbf{w}_{j,k,t} - \rho^{sw}\mathbf{w}_{j,k,t-1}) \end{aligned}$$

A.1.2 Other Institutional Details

There are additional institutional details that affect the estimation of the model. First, the PS2 was released in October 2000, and the Xbox and GC in November 2001.[4] I assume consumers knew the Xbox and GC would be released during the 2001 holiday season before the PS2 was released.[5] I model the consumer's relevant problem from October 2000 to October 2001 as a finite horizon optimal stopping problem with only one hardware console available, and assume that consumers know the starting lifetime expected utilities for the Xbox and GC.

Second, the PS2 was backwards compatible with titles released for Sony's previous generation console, the original Playstation (PS1). Any utility derived from titles released for the PS1 prior to October 2000 as well as expectations over future software availability are subsumed in the PS2's fixed-effect; however, any unexpected utility from PS1 titles released afterwards would not be accounted for. From the release of the PS2 in October 2000, there were 387 titles released for the PS1, 332 of which were not also released for the PS2. None were large successes. Since it is impossible to differentiate whether or not purchasers of these software titles owned a PS1, PS2, or both, I will assume that these titles do not influence a consumer's decision to purchase a PS2.

Third, the PS2 exhibited shortages during the first few months of its launch and supply was not able to meet demand; as a result, the model may potentially predict a lower expected lifetime

[4]Sega's Dreamcast was discontinued on January 31, 2001, and is not considered in this paper.

[5]Microsoft officially announced the Xbox on March 10, 2000 and Nintendo announced the GC on August 25, 2000, although their existence was rumored for months prior. As often is the case, console manufacturers announce the upcoming release of a new console far in advance to drum up support from software developers and interest from consumers.

utility for the PS2 than the true value for those early months. However, if access to the console was independent of consumer heterogeneity (and consumers purchased in the same proportion had there not been a shortage), then ignoring the implied $\delta_{i,j,t,\iota}$ for the first few months during estimation would still yield consistent estimates. This would be equivalent to removing the initial values of $\nu_{j,t}^{hw}$ for the PS2 when constructing moments; doing so did not alter results.

A.1.3 Further Comments on Identification

Given parametric and functional form assumptions, this section provides intuition for the identification of the variance of consumer heterogeneity (α_i^γ) and the complementarity between consoles (D) in simplified versions of the consumer demand model.

Consumer Heterogeneity. Assume $\beta = 0$ (no forward-looking behavior), and consider two periods with a single hardware platform delivering utility $u_{i,j,t}^{hw} = \delta_j + \Gamma_{j,t}(\alpha_i^\gamma) + \epsilon_{i,j,t}$, where $\Gamma_{j,t}(\alpha_i^\gamma)$ represents the option value of being able to purchase software for the platform and α_i^γ represents unobserved consumer heterogeneity. Each period there is one software product (indexed by t) which only exists for one period and which delivers utility $u_{i,t}^{sw} = \alpha_i^\gamma + \zeta_t + \epsilon_{i,t}^{sw}$. Assume ϵ shocks and the outside option for each product are drawn iid extreme value, and software shocks are observed only after purchasing a platform. This implies the option value of being able to purchase a software product in each period can be computed: $\Gamma_{i,j,t} = \ln(1 + \exp(\alpha_i^\gamma + \zeta_t))$.

Predicted hardware shares in each period are:

$$s_{j,t}^{hw} = \int \frac{\exp(\delta_j + \Gamma_{j,t}(\alpha_i^\gamma))}{1 + \exp(\delta_j + \Gamma_{j,t}(\alpha_i^\gamma))} dF_t(\alpha_i^\gamma)$$

and software shares are:

$$s_t^{sw} = \int \frac{\exp(\alpha_i^\gamma + \zeta_t)}{1 + \exp(\alpha_i^\gamma + \zeta_t)} dG_t(\alpha_i^\gamma)$$

where $F_t(\alpha_i^\gamma)$ is the distribution of heterogeneity in the population of those who have not purchased a console, and $G_t(\alpha_i^\gamma)$ is the distribution for those who purchased a console by period t; both can be computed from the model.

If $\alpha_i^\gamma \sim N(0, \sigma^\gamma)$, there are four parameters to estimate $(\{\delta_j, \{\zeta_t\}_{t=1,2}, \sigma^\gamma\})$ with four observed shares in the data to match $(\{s_{j,t}^{hw}, s_t^{sw}\}_{t=1,2})$. If $\sigma^\gamma = 0$, the model would predict $s_{j,1}^{hw} = s_{j,2}^{hw}$ if $s_1^{sw} = s_2^{sw}$ as $\Gamma_{j,t}$ would then be the same in each period; otherwise, no consumer heterogeneity can be rejected. Note observing an additional period t with another software release introduces 2 additional moments $(\{s_{j,t}^{hw}, s_t^{sw}\})$, but only one additional parameter to estimate (ζ_t); thus observing additional periods and software releases allows for the identification of α^Γ and other parameters.

Console Complementarity. Assume $\beta = 0$ and $\sigma^\gamma = 0$ (no consumer heterogeneity). Consider two periods with three hardware platforms $\{P, X, G\}$, with P active in both periods, and X and G active in the last. Consumers can only buy at most 1 platform per period.

Let $u_{i,j,t,\iota}^{hw} = \delta_j + \Gamma_{j,t}(\iota) + D_{\iota,t} + \epsilon_{i,j,t}$, where $D_{\iota,t} = D$ if $t = 2$ and the consumer purchased P in period 1; otherwise, $D_{\iota,t} = 0$. In period 2, there are 2 software products: product 1 is released only on platform X, and product 2 on P and G. Each title k delivers utility $u_k^{sw} = \zeta_k + \epsilon_{j,k,t}^{sw}$ if purchased. Again, assume ϵ shocks and the outside option for each product are drawn iid extreme value, and software shocks are observed only after purchasing a platform.

Software utilities are: $\Gamma_{P,1} = 0$, $\Gamma_{P,2} = \ln(1 + \exp(\zeta_2))$, $\Gamma_{X,2} = \ln(1 + \exp(\zeta_2))$, and $\Gamma_{G,2}(\iota) = \ln(1 + \exp(\zeta_2))\mathbf{1}_{\iota=\{0\}}$, where $\mathbf{1}_{\iota=\{0\}}$ is an indicator for a consumer not having purchased a console in period 1.

The model would predict the share of all consumers who purchase a console in a given period to be:

$$
\begin{aligned}
s_{P,1}^{hw} &= \frac{\exp(\delta_P)}{1+\exp(\delta_P)} \\
s_{P,2}^{hw} &= \left(1-s_{P,1}^{hw}\right)\frac{\exp(\delta_P+\Gamma_{P,2})}{1+\sum_{j=\{P,X,G\}}\exp(\delta_j+\Gamma_{j,2}(\{0\}))} \\
s_{X,2}^{hw} &= \left(1-s_{P,1}^{hw}\right)\left(\frac{\exp(\delta_X+\Gamma_{X,2})}{1+\sum_{j=\{P,X,G\}}\exp(\delta_j+\Gamma_{j,2}(\{0\}))}\right) \\
&\quad + \left(s_{P,1}^{hw}\right)\left(\frac{\exp(\delta_X+\Gamma_{X,2}+D)}{1+\exp(\delta_X+\Gamma_{X,2}+D)+\exp(\delta_G+D)}\right) \\
s_{G,2}^{hw} &= \left(1-s_{P,1}^{hw}\right)\left(\frac{\exp(\delta_G+\Gamma_{G,2}(\{0\}))}{1+\sum_{j=\{P,X,G\}}\exp(\delta_j+\Gamma_{j,2}(\{0\}))}\right) \\
&\quad + \left(s_{P,1}^{hw}\right)\left(\frac{\exp(\delta_G+D)}{1+\exp(\delta_X+\Gamma_{X,2}+D)+exp(\delta_G+D)}\right)
\end{aligned}
$$

and software shares for each title k onboard each platform to be:

$$
s_k^{sw} = \frac{\exp(\zeta_k)}{1+\exp(\zeta_k)}
$$

where the model would predict software shares onboard P and G for title 2 to be the same. Note $\{\zeta_1, \zeta_2\}$ are identified from software shares alone, and δ_P is identified from $s_{P,1}^{hw}$.

To understand how the remaining parameters $\{\delta_X, \delta_G, D\}$ can be identified from the remaining shares $\{s_{P,2}^{hw}, s_{X,2}^{hw}, s_{G,2}^{hw}\}$, assume for sake of argument that $\zeta_1 = \zeta_2$ so $\Gamma_{P,2} = \Gamma_{X,2} = \Gamma_{G,2}(\{0\})$. Compare $s_{X,2}^{hw}$ to $s_{G,2}^{hw}$: since $\Gamma_{X,2}$ influences new purchasers of X regardless of inventory, whereas $\Gamma_{G,2}$ does not, D influences how differences in $s_{X,2}^{hw}$ and $s_{G,2}^{hw}$ are attributable to differences in δ_X and δ_G (which would be the case if $D = -\infty$), and to the share of consumers who previously purchased P (given by $s_{P,1}^{hw}$). As such, different values of D implies different values of δ_X and δ_G, which in turn affects the ability of the model to match $s_{P,2}^{hw}$.

Moments based on matching the number of households who own at least one videogame console provide additional identifying restrictions.

A.2 Hardware-Software Network Formation

A.2.1 Computation of Profits

Rewrite (13) in the main paper as:

$$
E[\pi_k(\mathbf{s}_k;\theta_C)|\Omega_{r_k-\tau}] = \left(\sum_{t=r_k}^{T}\beta^{\tau+t-r_k}\sum_{j\in\mathbf{s}_k}E[M_{j,k,t}s_{j,k,t}((1-rmkup_t)p_{j,k,t}-mc_j)]\right) - C_k(\mathbf{s}_k;\theta_C)
$$

where $Q_{j,k,t}$ has been broken into: $s_{j,k,t}$, which represents the share of consumers who purchase title k, and $M_{j,k,t}$, which represents the number of consumers on platform j who have not yet purchased title k. $s_{j,k,t}$ is solely a function of $\zeta_{k,t}$ and the distribution of consumer types onboard platform j who have not yet purchased the title. If $IB_{j,t}$ is the number of consumers who own console j at time t, and $IB_{j,k,t}$ the number of consumers who own title k on platform j, then

$M_{j,k,t} = IB_{j,t} - IB^k_{j,t}$, where $IB^k_{j,t} = IB^k_{j,t-1} + M_{j,k,t-1}s_{j,k,t-1}$. From the demand side, recall a sufficient statistic for determining $IB_{j,t}$ is $\{\delta_{i,j,t,\iota}\}$.[6]

To form the first part of $E[\pi_k(\mathbf{s}_k); \theta_C]$, only expected values of $\{\{\delta_{i,j,t,\iota}\}, \{\zeta_{j,k,t}\}, \{p_{j,k,t}\}\}_{j\in\mathcal{J}_t, t>r_k-\tau}$ are initially required. I obtain these using a simulated frequency approach as in Pakes (1986): multiple sample paths of these variables are created via forward simulation using the estimated transition processes from the demand system, and the appropriate quantities $M_{j,k,t}$ and $s_{j,k,t}$ are calculated at each point in time. At release date r_k, the predicted hardware mean utilities $\{\delta_{i,j,t,\iota}\}$ are increased by the amount software k contributes to each platform it joins, as determined by its choice of strategy $\mathbf{s}_k$. To simulate each title's expected price path, I assume that each software title perceives that it follows a first-order Markov process (estimated from the data), and depends only on its own previous value and the month-of-year.

A.2.2 Computation of Equilibrium

Market equilibrium in each counterfactual is computed using the following algorithm:

1. Fix the transition processes governing the evolution of $\{\delta_{i,j,t,\iota}\}$ and $\{\zeta_{j,k,t}\}$ to starting beliefs $F^0 \equiv \{F^0_{i,\iota}\}$ and $G^0 \equiv \{G^0_{i,j}\}$. For robustness, I used 5 different sets of starting beliefs F^0 which govern the evolution of hardware qualities δ: one which assumes that no software title joins any console, one which all titles join every console, and three different sets in which all titles join only one console.

2. In each iteration n, I proceed forward from $t = 0$ and at every period: update $\{\delta^n_{i,j,t,\iota}\}$ for every console based on the set of new titles released and their chosen strategies; evaluate consumer demand over the set of hardware and software products; and compute the optimal strategy $\mathbf{s}^n_k$ for each title $k \in \mathcal{K}_{t+\tau}$ to be released τ months in the future.

3. After the optimal actions for all titles in every period are computed, I use the implied paths of $\{\delta^n_{i,j,t,\iota}\}$ and $\{\zeta^n_{j,k,t}\}$ to update the transition processes according to the regressions given by (8) and (10) in the main text, obtain new estimates for F^{n+1} and G^{n+1}, and repeat the simulation until no software title changes its chosen action from the previous iteration and the estimated transition processes converge.

B Model Fit: Consumer Beliefs and Prediction Errors

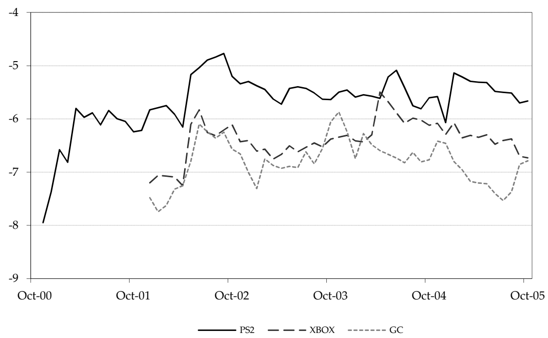
Figure 2a plots the seasonally adjusted predicted values of $\{\delta_{j,t}\}_{j\in\mathcal{J}_t,\forall t}$ for the mean consumer from the full demand model. Consumers' expectations of hardware mean-utilities are assumed to be a function of the previous values across all consoles as well as the time of year: as evident, values for each console do seem to track each other, and adjusting for seasonality dramatically smooths out seasonal spikes. Figure 2b plots the values of $\Gamma_{j,t}$ for a consumer in the 90th percentile of the distribution of α^γ_i who does not own any consoles. There are substantial seasonal spikes around the holiday months, and software utility gradually declines over time as older titles "decay" in quality (i.e., are no longer desirable) and fewer periods remain for new software releases. At the end of the sample, the Xbox and PS2 have the highest lifetime software utility, with the Xbox aided greatly by the release of the game *Halo 2* in November 2004 (which nearly 40% of all Xbox owners purchased).

To test how well the restriction on consumer beliefs approximates future values of hardware utilities $\{\delta_{i,j,t,\iota}\}$, Figure 2c plots the error in the realized value of $\delta_{i,j,t+1,\iota}$ from the expected value

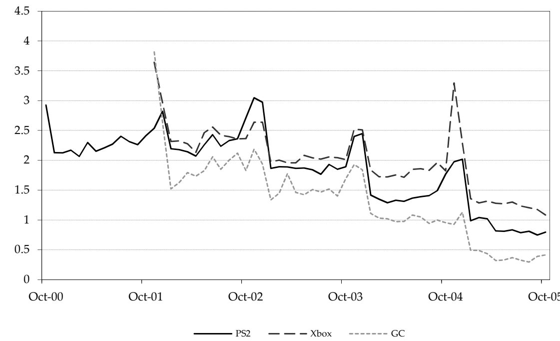
[6] Although for notational simplicity I have ignored consumer heterogeneity and inventory, I have controlled for them in estimation.

Figure 2: Fit of Model

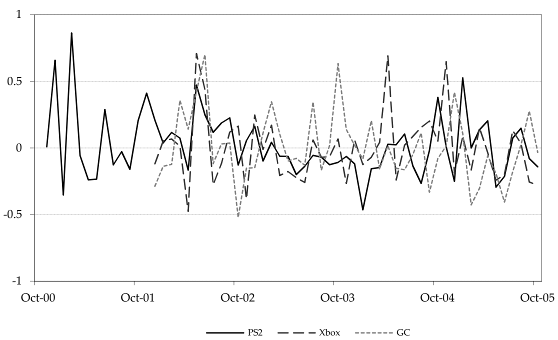
(a) $\{\delta_{i,j,t,\iota}\}_{\forall j,t}$ (Seasonally Adjusted)



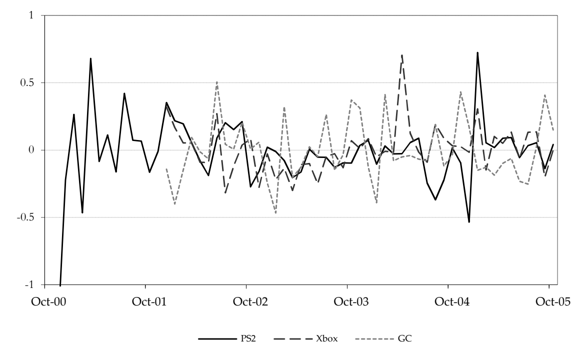
(b) $\{\Gamma_{j,t}(\alpha_i^\gamma; \iota = 0)\}_{\forall j,t}$



(c) $\delta_{i,j,t+1,\iota} - E(\delta_{i,j,t+1,\iota}|\{\delta_{i,j,t,\iota}\}_{\forall j}, m(t))$



(d) $\{\nu_{j,t}^{hw}\}_{\forall j,t}$



Notes: (a) Realized values of hardware mean-utility δ for the mean consumer who owns no consoles implied by the full demand model. Values are seasonally adjusted by the estimated month effects presented in Table 3.
(b) Realized values of software utility $\Gamma_{j,t}$ for consumer at the 90th percentile of the distribution of α_i^γ who owns no consoles implied by the full demand model.
(c) Errors between realized ($\delta_{i,j,t+1,\iota}$) and predicted values ($E(\delta_{i,j,t+1,\iota}|\{\delta_{i,j,t,\iota}\}_{\forall j}, m(t))$) of hardware mean-utility for mean consumer with no inventory using the estimated Markov transition process given by (9).
(d) Predicted residuals in hardware unobserved characteristics from full demand model: $\nu_{j,t}^{hw} \equiv \xi_{j,t} - \rho^{hw}\xi_{j,t}$.

implied by the estimated transition process $F_{i,\iota}(\{\delta_{i,j,t,\iota}\}, m(t))$ for a consumer t the 90th percentile of the distribution of α_i^γ with no inventory. Conditioning only on past values of $\{\delta_{i,j,t,\iota}\}$ and the month-of-year yields relatively small predicted errors: errors comprise on average only 3.3% of the absolute value of $\delta_{i,j,t+1,\iota}$, and are not serially correlated or appear to exhibit other time trends. Errors are substantially larger in magnitude without controlling for seasonality effects,

The key assumption used for inference is that changes in unobserved product characteristics $\xi_{j,t}$ and $\eta_{j,k,t}$ ($\nu_{j,t}^{hw}$ and $\nu_{j,k,t}^{sw}$) are independent and uncorrelated with a vector of moments (which include changes in observable product characteristics excluding price). Figure 2d plots the implied values of $\{\nu_{j,t}^{hw}\}$ for each hardware device. These values are not found to be serially correlated, and there do not seem to be any significant common shocks across platforms. From the demand estimates, there is significant persistence in hardware and software unobservables with ρ^{hw} estimated to be .565 and ρ^{sw} to be .695; thus approximately 32% and 48% of the variance in $\xi_{j,t}$ and $\eta_{j,k,t}$ are explained by their previous values. However, the variation from $\nu_{j,t}^{hw}$ only comprises $10-17\%$ of the total variance in $\delta_{j,t}$ across consoles, and thus does not necessarily indicate lack of explanatory power on the part of the model.

Finally, I also estimated the model assuming consumers had perfect expectations over future software availability $\tilde{\Lambda}^f$; though some parameter estimates were slightly changed, the economic predictions of the model were not significantly different.

C Alternative Specifications for Consumer Demand

Parameter estimates from alternative specifications of the demand system are presented in Table 1. Column (i) estimates a static model without consumer heterogeneity (i.e., a standard logit model) where any dynamic interdependencies across periods are removed (e.g., consumers do not leave the market, time their purchases, or account for future software releases) and unobservable product characteristics are not assumed to be persistent ($\rho^{hw} = \rho^{sw} = 0$); (ii) introduces dynamic considerations; (iii) introduces consumer heterogeneity; and (iv) adds multihoming, which is the full model presented in the main paper. Estimation of models (i) and (ii) without any consumer heterogeneity is equivalent to estimating the hardware and software side sequentially in two separate stages; the nested fixed point routine introduced in the previous section to handle consumer selection onto platforms is unnecessary. Specifications (i)-(iii) do not allow for multihoming, and do not use moments on the total number of households which own a console; as noted before, models in which consumers only buy one console can be rejected by the data since the number of households that own a console is less than the total number of consoles sold. In specifications (iii) and (iv), heterogeneity in price sensitivity was not found to be statistically significant, and $\sigma^{p,hw} = \sigma^{p,sw} = 0$ for the reported results.

Estimated Price Elasticities Table 2 reports own and cross-price elasticities for platforms across the four specifications, where each cell reports the percent change in sales of the platform located in the column due to a permanent 1% increase in the price of the row-platform, and "Outside" indicates substitution to the outside good. Since platforms are active for multiple periods, the price change is assumed to apply across the entire time period, and market shares are computed from installed base figures at the end of the sample period. Specification (iv) predicts that platforms would see their installed base fall approximately 1.4 to 2.0% from a 1% price increase; cross price elasticities are smaller in magnitude, with a 1% increase in price of the PS2 increasing sales of the Xbox and GC by approximately .1%. Most consumers who substitute away from a console following a price increase opt to consume the outside good rather than purchase another console.

Turning to specification (i), as the estimated $\alpha_0^{p,hw}$ in Table 1 is -0.005 as opposed to -0.013 in the full model, it follows that estimated price elasticities are closer to 0 as well. This is primarily due to two effects: first, as the static model (i) fails to account for the durability of goods and consumers leaving the market, the predicted share of consumers purchasing a hardware device falls faster than had durability been accounted for, which in turn makes it appear consumers are not particularly responsive to prices falling over time (c.f. Aguirregabiria and Nevo (2012)). Secondly, a static model doesn't allow for consumers to anticipate future software releases, and hence has a difficult time explaining why consumers purchase early in spite of high prices without biasing price sensitivities to 0. Specifications (ii) and (iii) yield statistically equivalent estimates $\alpha_0^{p,hw}$, but are half the magnitude of the full model and predict more inelastic price responses to hardware.

Table 3 reports software price elasticities, and provides the percentage change in sales for a representative hit title on each platform following a permanent 1% price increase. These titles are the most popular titles on each console released in the first year of a console's existence (which all happened to be exclusive); as such, they were released both when they might have had the largest effect on eventual hardware adoption, and when selection by consumers onto platforms might have

Table 1: Estimated Parameters of Demand System

		No Consumer Heterogeneity				Consumer Heterogeneity			
		Static Model Singlehoming (i)		Dynamic Model Singlehoming (ii)		Dynamic Model Singlehoming (iii)		Dynamic Model Multihoming (iv)	
	Variable	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.
Nonlinear Parameters	β			0.270***	0.008	0.000	0.001	0.934***	0.021
	ρ^{hw}			0.668***	0.015	0.520***	0.018	0.619***	0.024
	ρ^{sw}			0.663***	0.001	0.698***	0.001	0.695***	0.002
	σ^{γ}					2.738***	0.286	1.939***	0.139
	α^{Γ}	1.170***	0.325	1.371***	0.228	0.216***	0.075	0.663***	0.204
	D							0.000	0.466
Hardware Parameters	$\alpha_0^{p,hw}$	-0.005***	0.001	-0.006**	0.002	-0.006***	0.002	-0.013***	0.003
	d_{PS2}	-4.766***	0.384	-4.218***	0.784	-4.027***	0.619	-1.902**	0.869
	d_{XBOX}	-5.544***	0.330	-5.034***	0.689	-4.904***	0.540	-3.349***	0.769
	d_{GC}	-6.221***	0.276	-5.745***	0.561	-5.664***	0.447	-4.399***	0.635
	age	0.016*	0.009	-0.005	0.016	0.005	0.013	-0.036**	0.017
	age^2	0.000***	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Software Parameters	$\alpha_0^{p,sw}$	-0.035***	0.001	-0.035***	0.003	-0.037***	0.003	-0.040***	0.003
	age	-0.232***	0.002	-0.201***	0.004	-0.180***	0.005	-0.183***	0.005
	age^2	0.002***	0.000	0.001***	0.000	0.001***	0.000	0.001***	0.000
GMM Objective		1398.917[(a)]		367.195[(a)]		258.894[(a)]		259.567	
# HW Obs. (n^{hw})		151		151		151		151	
# SW Obs. (n^{sw})		44209		44209		44209		44209	

Notes: β is the discount factor; ρ^{hw} and ρ^{sw} are the estimated coefficients on the autoregressive processes for $\xi_{j,t}$ and $\eta_{j,k,t}$; σ^{γ} is the standard deviation of consumer heterogeneity for gaming intensity α^{γ}; α^{Γ} is the coefficient on software utility; $\alpha_0^{p,hw}$ and $\alpha_0^{p,sw}$ are price sensitivity coefficients; D is the hardware complementarity term. For the remaining hardware and software coefficients, d_j are fixed effects for platform j, and age and age^2 are monthly decay effects. ***, **, and * indicate significance at the 1%, 5% and 10% levels.
[(a)] All singlehoming models do not utilize the additional moment restricting the number of households who purchase a console to be less than 44.1M, and hence the objective does not contain the penalty.

been seen as most severe. Magnitudes are similar across platforms.

Hardware Responsiveness to Software Table 4 presents changes in hardware installed bases if the same three hit titles used to compute software price elasticities were not available on each console. There are substantive differences across specifications. First, models (i) and (ii) dramatically overstate substitution to the outside good in response to changes in software availability. This is due to the failure of these specifications to control for consumer heterogeneity; since consumers who purchase a console are predisposed to gaming, they are more likely to purchase another console than purchase none at all if a title is lost. This key difference between those who own a videogame console and those who do not will prove crucial to control for when evaluating the counterfactual of universal compatibility in the next section. Second, specification (iii) overpredicts cross-console elasticities since consumers are not allowed to multihome.

Forced Compatibility Table 5 reports results across specifications (i)-(iv) of the model. The full specification (iv) predicts if all software titles were forced to multihome, consumers are substantially better off: approximately 5M more households purchase consoles, and total software sales increase

Table 2: Estimated Hardware Own and Cross-Price Elasticities

		PS2	XBOX	GC	Outside
Model (i)	PS2	-1.037	0.009	0.009	0.666
Static		*(-1.613, -0.758)*	*(0.006, 0.013)*	*(0.007, 0.015)*	*(0.487, 1.036)*
No Multihoming	XBOX	0.004	-0.903	0.005	0.287
No Heterogeneity		*(0.003, 0.007)*	*(-1.406, -0.660)*	*(0.004, 0.009)*	*(0.210, 0.447)*
	GC	0.002	0.003	-0.626	0.146
		(0.002, 0.004)	*(0.002, 0.004)*	*(-0.975, -0.457)*	*(0.107, 0.228)*
Model (ii)	PS2	-1.014	0.280	0.269	0.413
Dynamic		*(-1.677, -0.280)*	*(0.077, 0.462)*	*(0.074, 0.445)*	*(0.114, 0.684)*
No Multihoming	XBOX	0.073	-0.989	0.089	0.173
No Heterogeneity		*(0.020, 0.121)*	*(-1.634, -0.273)*	*(0.024, 0.147)*	*(0.048, 0.286)*
	GC	0.040	0.052	-0.704	0.086
		(0.011, 0.066)	*(0.014, 0.086)*	*(-1.165, -0.194)*	*(0.024, 0.142)*
Model (iii)	PS2	-1.014	0.303	0.299	0.403
Dynamic		*(-1.547, -0.541)*	*(0.151, 0.462)*	*(0.146, 0.453)*	*(0.217, 0.615)*
No Multihoming	XBOX	0.081	-0.997	0.101	0.168
Heterogeneity		*(0.040, 0.124)*	*(-1.520, -0.528)*	*(0.048, 0.153)*	*(0.091, 0.257)*
	GC	0.045	0.058	-0.709	0.083
		(0.022, 0.068)	*(0.028, 0.088)*	*(-1.083, -0.376)*	*(0.045, 0.127)*
Model (iv)	PS2	-1.973	0.148	0.061	0.695
Dynamic		*(-2.714, -1.347)*	*(-0.162, 0.456)*	*(-0.222, 0.360)*	*(0.480, 0.974)*
Multihoming	XBOX	0.032	-2.004	0.048	0.238
Heterogeneity		*(-0.040, 0.108)*	*(-2.738, -1.373)*	*(-0.022, 0.131)*	*(0.153, 0.347)*
	GC	0.011	0.019	-1.432	0.116
		(-0.024, 0.050)	*(-0.018, 0.068)*	*(-1.967, -0.982)*	*(0.074, 0.172)*

Notes: Cell entries i, j, where i indexes row and j indexes column, provides the percent change in quantity sold with a permanent 1% increase in the price of console i (where *Outside* represents non-purchasers). 95% confidence intervals are provided in parenthesis below estimates.

Table 3: Estimated Software Own-Price Elasticities

	DYN	HET	MH	PS2	XBOX	GC
Model (i)	No	No	No	-1.211	-1.040	-0.849
				(-1.273, -1.127)	*(-1.093, -0.968)*	*(-0.893, -0.790)*
Model (ii)	Yes	No	No	-1.208	-1.047	-0.856
				(-1.369, -0.989)	*(-1.186, -0.857)*	*(-0.969, -0.700)*
Model (iii)	Yes	Yes	No	-1.082	-0.988	-0.883
				(-1.222, -0.893)	*(-1.107, -0.823)*	*(-0.972, -0.757)*
Model (iv)	Yes	Yes	Yes	-1.275	-1.144	-0.958
				(-1.435, -1.101)	*(-1.290, -0.975)*	*(-1.083, -0.814)*

Notes: Percentage change in total quantity sold of a top selling title on each console conditional on a permanent 1% increase in the price of that title (where *Outside* represents non-purchasers). The software titles are *Grand Theft Auto III* for the PS2, *Halo* for the Xbox, and *Super Smash Bros.* for the GC. 95% confidence intervals are provided in parenthesis below estimates.

by almost 400M units. Through this, consumers are predicted to gain \$1.8B in welfare primarily due to the accessibility of more titles onboard the systems they purchase. This represents 6% of total hardware and software revenues during this 5 year period.

The differences in counterfactual predictions across alternative model specifications (i)-(iii) are

Table 4: Hardware Elasticities from Losing A Top Title

		PS2	XBOX	GC	Outside
Model (i)	PS2	-1.390	0.016	0.018	0.896
Static		*(-2.028, -0.634)*	*(0.008, 0.024)*	*(0.008, 0.027)*	*(0.409, 1.307)*
No Multihoming	XBOX	0.027	-4.290	0.035	1.366
No Heterogeneity		*(0.013, 0.037)*	*(-5.965, -2.079)*	*(0.017, 0.049)*	*(0.662, 1.899)*
	GC	0.021	0.018	-3.383	0.790
		(0.010, 0.028)	*(0.009, 0.024)*	*(-4.621, -1.680)*	*(0.392, 1.079)*
Model (ii)	PS2	-1.776	0.617	0.608	0.677
Dynamic		*(-2.220, -1.394)*	*(0.485, 0.770)*	*(0.477, 0.759)*	*(0.532, 0.847)*
No Multihoming	XBOX	0.748	-6.968	0.880	1.065
No Heterogeneity		*(0.609, 0.897)*	*(-8.414, -5.641)*	*(0.716, 1.056)*	*(0.860, 1.291)*
	GC	0.368	0.444	-5.015	0.558
		(0.300, 0.441)	*(0.361, 0.534)*	*(-6.062, -4.057)*	*(0.450, 0.677)*
Model (iii)	PS2	-0.835	0.635	0.722	0.165
Dynamic		*(-1.004, -0.356)*	*(0.179, 0.824)*	*(0.189, 0.959)*	*(0.112, 0.169)*
No Multihoming	XBOX	0.680	-3.699	0.923	0.341
Heterogeneity		*(0.201, 0.933)*	*(-4.564, -1.541)*	*(0.257, 1.216)*	*(0.206, 0.359)*
	GC	0.336	0.416	-2.597	0.171
		(0.101, 0.447)	*(0.126, 0.518)*	*(-3.136, -1.109)*	*(0.107, 0.181)*
Model (iv)	PS2	-0.742	0.357	0.143	0.156
Dynamic		*(-0.952, -0.446)*	*(0.141, 0.580)*	*(0.009, 0.289)*	*(0.099, 0.205)*
Multihoming	XBOX	0.291	-5.453	0.731	0.198
Heterogeneity		*(0.092, 0.502)*	*(-6.670, -3.686)*	*(0.381, 1.017)*	*(0.113, 0.264)*
	GC	0.046	0.092	-3.750	0.089
		(-0.001, 0.104)	*(0.039, 0.150)*	*(-4.505, -2.628)*	*(0.054, 0.120)*

Notes: Cell entries i, j, where i indexes row and j indexes column, provides the percentage change in sales of console j upon console i losing a top software title (where *Outside* represents non-purchasers). The software titles are *Grand Theft Auto III* for the PS2, *Halo* for the Xbox, and *Super Smash Bros.* for the GC. 95% confidence intervals are provided in parenthesis below estimates.

stark. In particular, models (i) and (ii) which do not account for consumer heterogeneity predict that under universal compatibility nearly all non-purchasers become hardware purchasers—i.e., specification (i) predicts (more) than all households now purchase a console; in specification (ii), an additional 46M households purchase. Since both specifications fail to account for the fact that households which did not previously purchase a console are less predisposed to gaming, and hence are less responsive to software availability, they vastly overstate gains from universal compatibility: consumer welfare would increase by \$10-15B (approximately half of total industry revenues from this period). Such predictions are unrealistic.

Introducing consumer heterogeneity in (iii) helps to correct for this out-of-sample prediction, but generally underestimates the impact of software availability on hardware sales: e.g., it predicts an additional 3M consumers would purchase consoles in the event of universal compatibility and a consumer welfare increase of \$.8B. Additionally, failing to account for consumer multihoming underestimates the harm borne by the Xbox and GC under this counterfactual: in particular, a singlehoming model fails to account for the fact that consumers substitute from purchasing multiple consoles to only purchasing the PS2.

Table 5: Hardware Elasticities from Universal Compatibility of Software Titles

	DYN	HET	MH	PS2	XBOX	GC	Outside
Model (i)	No	No	No	123.125	75.901	92.047	-126.751
				(30.271, 293.109)	*(25.332, 133.110)*	*(29.056, 164.987)*	*(-273.442, -34.844)*
Model (ii)	Yes	No	No	157.364	-19.664	27.006	-81.768
				(104.636, 199.544)	*(-50.618, 6.563)*	*(3.701, 37.256)*	*(-92.625, -62.100)*
Model (iii)	Yes	Yes	No	6.583	2.434	3.688	-4.604
				(4.518, 8.420)	*(1.973, 3.289)*	*(2.805, 4.616)*	*(-5.911, -3.337)*
Model (iv)	Yes	Yes	Yes	19.213	-12.862	-7.823	-8.593
				(15.669, 24.477)	*(-14.754, -9.188)*	*(-9.734, -4.885)*	*(-10.993, -6.582)*

Notes: Percentage change in sales of each console subject to every software title multihoming and joining all three consoles. 95% confidence intervals are provided in parenthesis below estimates.

D Additional Robustness Tests for Consumer Demand

D.1 Pricing Instruments

Table 7 provides first stage regressions of hardware and software prices—both in levels and first-differences—on excluded instruments, which include lagged one and two-period prices, and current and lagged values of: Japan-U.S. exchange rates, the average price of other games of the same age released in prior months ($\tilde{p}^1$), and the average price of all games on other consoles in different genres ($\tilde{p}^2$). In all regressions, age and age squared terms and month dummies are used as controls; product fixed effects are used in level regressions. With the exception of first-differences in hardware prices, instruments are not found to be "weak" (c.f. Stock and Yogo (2005)); omitting moments in first differences for hardware did not substantially change results.

Results from failing to instrument for prices are reported in Table 6(v). Hardware price elasticities are not significantly different from those obtained with instruments. On the other hand, software price sensitivities are biased away from 0 without instrumenting for price: estimates of $\alpha_0^{p,sw}$ nearly double from $-.040$ to $-.071$, with software price elasticities increasing from (-1.3, -1.1, -0.96) across the three platforms to (-2.2, -2.0, -1.7). Possible explanations for the bias include price drops being correlated with demand shocks such as advertising campaigns and/or unanticipated high demand (which would occur if larger than expected sales causes higher desirability for the product (e.g., through direct network effects) and if firms engage in price skimming and reduce prices for durable software as high valuation consumers purchase and leave the market (c.f. Nair (2007))).

D.2 Consumer Myopia and Non-Forward Looking Behavior

In the model, β is identified via the impact of future software availability on current hardware sales. I have also estimated the model fixing β at different values. Results from fixing $\beta = .99$ do not yield significantly different results.

Table 6 (vi) reports results when β is fixed at 0—i.e., consumers do not time purchases, nor anticipate future software availability when purchasing hardware. Table 6 (vii) reports results when consumers do care about future software utility (hence, $\beta > 0$ and is freely estimated), but are assumed to not anticipate future utility when timing their purchases. However, products are still durable and all other dynamic considerations are still controlled for.

Table 6: Robustness Tests: No Pricing Instruments, Consumer Myopia, Software Independence

		No Price Inst. (v)		Myopic (vi)		No Purchase Timing (vii)		Adj. IB (viii)	
	Variable	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.
Global Parameters	β	0.934[(a)]		0.000[(b)]		0.955***	0.001	0.924***	0.023
	ρ^{hw}	0.643***	0.036	0.596***	0.018	0.592***	0.017	0.640***	0.029
	ρ^{sw}	0.796***	0.004	0.701***	0.001	0.697***	0.002	0.698***	0.002
	σ^{γ}	2.495***	0.101	3.077***	0.227	2.179***	0.073	2.111***	0.130
	α^{Γ}	0.555***	0.099	0.760***	0.187	0.802***	0.256	0.678***	0.200
	D	0.000	0.578	-0.001	1.664	0.016	0.862	0.000	0.490
Hardware Parameters	$\alpha_0^{p,hw}$	-0.012***	0.001	-0.006**	0.003	-0.009***	0.002	-0.013***	0.003
	d_{PS2}	-2.183***	0.377	-4.155***	0.873	-3.507***	0.725	-2.069**	0.901
	d_{XBOX}	-3.607***	0.340	-5.264***	0.676	-4.693***	0.616	-3.520***	0.800
	d_{GC}	-4.605***	0.323	-5.982***	0.548	-5.515***	0.501	-4.525***	0.661
	age	-0.031**	0.014	0.001	0.022	-0.005	0.016	-0.034*	0.018
	age^2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Software Parameters	$\alpha_0^{p,sw}$	-0.071***	0.001	-0.042***	0.003	-0.040***	0.003	-0.040***	0.003
	age	-0.187***	0.004	-0.187***	0.005	-0.182***	0.005	-0.181***	0.005
	age^2	0.001***	0.000	0.001***	0.000	0.001***	0.000	0.001***	0.000
GMM Objective		127.863		260.483		256.296		259.761	
# HW Obs. (n^{hw})		151		151		151		151	
# SW Obs. (n^{sw})		44209		44209		44209		44209	

Notes: Re-estimating the full dynamic demand model (specification (iv) in Table 1) without pricing instruments in (v), fixing $\beta = 0$ in (vi), assuming consumers don't time their purchases in (vii), and controlling for purchases of the same title across different platforms in (viii). ***, **, and * indicate significance at the 1%, 5% and 10% levels. [a, b] β was fixed in estimation to be either .934 or 0.

There are two main differences to focus on in these specifications. First, hardware price sensitivities are biased to 0 with own- and cross-price elasticities being approximately halved. This bias for (vi) is similar to that discussed in the previous subsection—i.e., controlling for future software is required to explain why consumers purchase hardware when prices are high when consumers are still price sensitive. For (vii), failing to incorporate forward looking behavior understates the degree of selection onto platforms, attributing non-purchase upon price decreases to price insensitivity.

Second, under the full-compatibility counterfactual, these specifications understate the degree to which market tipping occurs: assuming consumer myopia predicts that the PS2 would only gain 6% (as opposed to 19%) of new users, whereas the XBox and GC would lose only 6% and 3% (as opposed to 13% and 8%) of purchasers. Nonetheless, fixing $\beta = 0$ predicts approximately 4.3% more consumers would purchase a console under full compatibility yielding consumer welfare gains of \$1.8B, which is not statistically different. The model without forward looking behavior is broadly similar. Thus, the direction of market concentration appear to be robust; however, ignoring consumer expectations over future software availability or the timing of purchases understates the magnitude of tipping that would occur.

D.3 Software Independence

Substitution Across Platforms. The data provides the quantity of title k sold on console j at time t, denoted $q^o_{j,k,t}$; from this, the observed share of purchasers for title k, $s^o_{j,k,t} = q_{j,k,t}/(IB_{j,t} - \sum_{\tau<t}(q_{j,k,\tau}))$ (where $IB_{j,t}$ is the installed base of console j at time t) is constructed. The demand model predicts the share of consumers of type i who purchase title k to be $\hat{s}_{i,j,k,t}$ in

Table 7: Pricing Instruments

	price_t^{hw}		$\Delta\text{price}_t^{hw}$		price_t^{sw}		$\Delta\text{price}_t^{sw}$	
	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.
Instruments								
price_{t-1}	0.900***	0.080			0.745***	0.005		
price_{t-2}	-0.040	0.081	-0.126***	0.041	0.009**	0.005	-0.071***	0.002
e_t	1.012***	0.365	0.933***	0.360	0.282***	0.031	0.124***	0.032
e_{t-1}	-1.191***	0.350	-1.119***	0.346	-0.104***	0.031	-0.131***	0.030
$\tilde{p}_t^1$					0.005	0.008	0.009	0.008
$\tilde{p}_{t-1}^1$					0.023***	0.009	0.000	0.008
$\tilde{p}_t^2$					-0.097***	0.035	-0.109***	0.036
$\tilde{p}_{t-1}^2$					0.048*	0.036	0.105***	0.036
Controls								
Product FE	X				X			
Age, Age^2	X		X		X		X	
Month Dummies	X		X		X		X	
F-stat	126.7		7.2		9594.3		1014.1	
R^2	0.771		0.116		0.635		0.115	
# Obs.	151		151		44209		44209	

Notes: First stage regressions of hardware and software prices (in both levels and first differences (Δ)) on current and lagged values of: prices, Japan-U.S. exchange rates (e), the average price of other games of the same age released in prior months ($\tilde{p}^1$), and the average price of all games on other consoles in different genres ($\tilde{p}^2$).

(28), which is used to construct the predicted quantity of title k on console j sold at time t: $\hat{q}_{j,k,t} = \sum_i \hat{s}_{i,j,k,t}\tilde{IB}^{sw}_{j,k,t}$, where $\tilde{IB}^{sw}_{i,j,k,t} = \tilde{IB}_{i,j,t} - \sum_{\tau<t}(q_{i,j,k,\tau}$ is the potential installed base of users of type i who still may purchase title k, and $\tilde{IB}_{i,j,t}$ represents the potential marketsize of consumers of type i who may still purchase title k at time t. In the model used in the main paper, $\tilde{IB}_{i,j,t}$ is assumed to be all consumers of type i who have purchased console j by time t.

The main model does not impose any restrictions on purchases of the same software title across two different platforms. I estimate a variant of the model which attempt to control for some of the potential biases that may be introduced. I assume a consumer is equally likely to purchase the game on any system she owns.[7] This changes the potential marketsize used in estimation for each title k depending on the set of consoles k is released on; there are 4 cases to consider:

1 Title k is exclusive to j: The relevant installed base of hardware users (denoted by $\tilde{IB}^0_{i,j,t}$) used to construct predicted shares and quantities is unchanged from the main model;

2 Title k is onboard j and rival j': If consumers only purchase a title on the first platform they own, then $\tilde{IB}^2_{i,j,t}$ only includes consumers who purchase j before console j'. If consumers are assumed to purchase shared titles with equal probability across consoles, then (i) if a consumer purchases j and does not own j', $\tilde{IB}^2_{i,j,t}$ increases by 1; (ii) if a consumer owns j' and purchases j, $\tilde{IB}^2_{i,j,t}$ increases by 1/2; and (iii) if a consumer already owns j but then purchases j', $\tilde{IB}^2_{i,j,t}$ decreases by 1/2.

3 Title k is onboard j and rival j'': Same as case 2 (with j'' replacing j').

[7] A version of the model in which a consumer only purchases a version of a game on the first system she buys does not affect the main results of the paper.

4 Title k is onboard j and both rivals: If consumers only purchase a title on the first platform they own, $\tilde{IB}^4_{i,j,t}$ only increases if j is the first platform purchased. If consumers equally purchase consoles across platforms, then (i) $\tilde{IB}^4_{i,j,t}$ increases by 1 if j is the first console purchased, 1/2 if j is the second console purchased, and 1/3 if j is the third console purchased; (ii) $\tilde{IB}^4_{i,j,t}$ decreases by 1/2 if j is the only console owned and another console is purchased; and (iii) $\tilde{IB}^4_{i,j,t}$ decreases by 1/6 if j is one of two consoles owned, and a third console is purchased.

Note that the variant of the model which assumes consumers are equally likely to purchase a multihoming title across platforms is only an approximation; it cannot distinguish whether someone that multihomes has or hasn't already purchased a title, and is implicitly assuming that she has not. Nonetheless, it serves as a useful robustness check. Results are reported in Table 6 (viii). Parameter estimates and counterfactual predictions are not found to be statistically different from the estimated model.

Substitution on the Same Platform. To test the restriction that software titles are independent on the same platform, I re-estimate the model with additional variables in software characteristics α^w in an attempt to control for potential software substitutability. In particular, I include the number of titles released in a given month on the same platform, both in the same genre as well as across all genres; I also include of the number of other "hit" titles (defined as selling $> 1M$ copies) released in a given month on a platform, interacted with whether or not the title of interest was also a hit title. If software substitutability were significant, one would expect that the number of games released onboard a system (either overall or in the same genre) would impact software demand.

Table 8 reports results. With the exception of Fighting and Racing games on the PS2, absolute magnitudes of all significant coefficients are small—i.e., an additional title released in the same genre onboard the same platform is equivalent to less than a \$1 impact in perceived price for that game. For Fighting and Racing games onboard the PS2, coefficients are positive, which indicate an additional game released in the same genre is equivalent to an approximate \$3 *decrease* in price; given the slightly different estimated software month from those in the full model and the strong seasonality in software title releases (see Figure 1(d)), it is likely that such positive coefficients on software are picking up variation in seasonality effects across years and not necessarily software complementarity effects per se. Estimates also indicate that "hit" titles are not greatly affected by the release of other hit titles: i.e., another hit title on the PS2 released in the same month would have the same impact on another hit title as a \$0.50 increase in its price, which in turn would impact sales by approximately 0.6% (using own-price elasticity estimates from Table 3).

Table 8: Robustness Tests: Testing Software Independence

		Variable	Estimate	s.e.
Global Parameters		β	0.934[(a)]	
		ρ^{hw}	0.618***	
		ρ^{sw}	0.714***	
		σ^{γ}	1.939***	
		α^{Γ}	0.663***	
		D	0.000	
# of Titles Released By Genre	PS2	Family	0.013**	
		Shooter	0.015***	
		Action	0.048	
		Sports	0.003*	
		Racing	0.103***	
		Fighting	0.098***	
		Platformer	-0.003	
		RPG	-0.013***	
		Other	-0.001	
		Total	-0.001	
	XB	Family	-0.004	
		Shooter	-0.005	
		Action	-0.006	
		Sports	-0.006**	
		Racing	0.005	
		Fighting	0.002***	
		Platformer	0.027*	
		RPG	-0.004	
		Other	0.000	
		Total	-0.001	
	GC	Family	-0.001	
		Shooter	-0.003	
		Action	-0.015**	
		Sports	-0.027***	
		Racing	-0.002	
		Fighting	-0.001	
		Platformer	0.000	
		RPG	0.032***	
		Other	-0.006*	
		Total	0.001	
# of Hit Titles Released	PS2	isHit	-0.020***	
		NotHit	0.038***	
	XB	isHit	-0.006	
		NotHit	-0.010	
	GC	isHit	-0.009*	
		NotHit	0.017***	
		GMM Obj.	275.239[(b)]	

Notes: Re-estimating the full dynamic demand model (specification (iv) in Table 1), using the number of other titles released in a given month on a console in the same genre as well as across all genres (Total); "# of Hit Titles" is the number of software titles which sold over 1M copies released, which is interacted with whether or not the title of interest was a hit (isHit) or not (NotHit). ***, **, and * indicate significance at the 1%, 5% and 10% levels.

[a] β was fixed in estimation.

[b] The GMM objective is not directly comparable to those in Table 1 (iv) since the GMM weighting matrix is different given the additional variables contained within α^{w}.

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Erratum for *Vertical Integration and Exclusivity in Platform and Two-Sided Markets*, AER 2013

There is a typographical error on page 2974 of the published version of the article. Lines 4-5 of equation (11) should be:

$$= \ln\Bigg(\sum_{j' \notin \iota_{i,t}} \big(\exp(\delta_{i,j',t,\iota} + \beta E[EV_i(\{\delta_{i,j,t+1,\iota}\}_{j \in \mathcal{J}_{t+1}, \iota \in \mathcal{I}}, \iota_{i,t} \cup \{j'\},$$
$$m(t+1) | \{\delta_{i,j,t,\iota}\}_{j \in \mathcal{J}_t, \iota \in \mathcal{I}})])\big)$$

and not

$$= \ln\Bigg(\sum_{j' \notin \iota_{i,t}} \big(\exp(\delta_{i,j',t,\iota} + \beta E[EV_i(\{\delta_{i,j,t+1,\iota}\}_{j \in \mathcal{J}_{t+1}, \iota \in \mathcal{I}}, \cup \{j'\},$$
$$\iota_{i,t}, m(t+1) | \{\delta_{i,j,t,\iota}\}_{j \in \mathcal{J}_t, \iota \in \mathcal{I}})])\big)$$