In all the $\varepsilon$-$\delta$ proofs we did last week, we were able to find the exact value of the largest possible $\delta$ for any given $\varepsilon$. However, the algebra does not always work out so cleanly, as the next example demonstrates.

**Example 4.** $f(x) = \sqrt{x}$

We claim that $\lim_{x \to 16} f(x) = 4$.

**Scratch Work.**

We must find a $\delta > 0$ such that the following is true.

If , then .

Therefore, choosing $\delta = \ldots$ should work.

**Claim.** If $f(x) = \sqrt{x}$, then $\lim_{x \to 16} f(x) = 4$.

**Proof.** Let $\varepsilon > 0$ be given.

We must find a $\delta > 0$ such that if then .

We choose $\delta = \ldots$. 
Proof by Contradiction. When it is necessary to show that a function has no limit at a given $x$-value (as some of you may do in your presentations), it is often necessary to use the method known as “proof by contradiction.”

In this method, in order to prove a statement $P$ is true, we begin by assuming its negation (“not $P$”) is true and showing that this assumption leads to a contradiction. We then conclude that “not $P$” is not true; therefore, $P$ must be true, which is what we hoped to prove.

We now look at one famous example of this method of proof, which will require a few definitions and one lemma (which will help us prove our theorem).

Definition. An integer $k$ is even if $k = 2m$ where $m$ is an integer.

Definition. An integer $k$ is odd if $k = 2m + 1$ where $m$ is an integer.

Definition. A real number $r$ is rational if $r = \frac{p}{q}$ where $p$ and $q$ are integers (and can be written in “lowest terms” so that $p$ and $q$ share no common factor other than 1) and $q$ is not zero.

Lemma. If $n^2$ is even, then $n$ is even.

Proof.

Theorem. The number $\sqrt{2}$ is not rational.

Proof.

We will do a proof by contradiction.

We assume that $\sqrt{2}$ is rational.

Next Week. You will have time to work on your presentations and ask me questions.