Explicit simulations of convectively coupled waves on an equatorial β-plane

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Abstract

We present equatorial $\beta$-plane simulations of convectively coupled waves using an approach proposed by Kuang et al. (2005). The simulations are considered explicit in the sense that no convective parameterization is used. The simulated spectra and the horizontal and vertical structures of Kelvin, mixed Rossby-gravity (MRG), and eastward inertial gravity (EIG) waves are presented. The simulations reproduce the basic features of the observed waves well. An additional experiment was conducted to assess the sensitivity of the results to our simulation approach. Based on these results, we conclude that the approach proposed by Kuang et al. can capture the basic features of convectively coupled equatorial waves, and suggest it as a useful methodology for further investigations of these waves.
1. Introduction

Convectively coupled tropical waves were first identified from satellite cloud imagery more than three decades ago (Chang, 1970). Owing to the many observational studies since then, the main characteristics of these waves are now quite well documented (Takayabu, 1994; Wheeler and Kiladis, 1999; Wheeler et al., 2000; Straub and Kiladis, 2003). Theoretical efforts to explain the existence and characteristics of these waves have suggested a rich variety of mechanisms: excitation by extratropical eddies (Zhang and Webster, 1992), wave-conditional instability of the second kind (wave-CISK) (Lindzen, 1974), surface heat flux feedback (Emanuel, 1987; Neelin et al., 1987), frictional convergence (Wang, 1988), radiation feedback (Fuchs and Raymond, 2005), stratiform instability (Mapes, 2000; Majda and Shefter, 2001), and moisture feedback (Khouider and Majda, 2005).

While one could evaluate these ideas in terms of their theoretical foundation (Emanuel et al., 1994) and by comparing them with observations (Straub and Kiladis, 2003), much more can be learned from a comprehensive numerical model that can faithfully reproduce the observed waves. Such a model would provide data that are difficult to obtain from observations and would also facilitate perturbation experiments where various mechanisms are removed to evaluate their importance. The standard tool for modeling the large-scale circulation, the General Circulation Model (GCM), however, does not simulate these waves well. As summarized in Lin et al. (2006), the convectively coupled waves simulated by GCMs are in general too weak and have excessively large equivalent depths. Moreover, since the essence of convectively coupled waves is the interaction between the large-scale circulation and deep convection, models that parameterize this
interaction, such as GCMs, do not serve as an independent testing tool. Models that can explicitly simulate this interaction (i.e., without cumulus parameterizations) are needed. Because of the large separation in scale between deep cumulus convection and the large-scale circulation, three-dimensional simulations that explicitly simulate all relevant scales are very expensive computationally (Tulich et al., 2006). With advances in computational capacity, global 3D nonhydrostatic model simulations with a resolution capable of simulating deep cumulus convection explicitly have started to appear (Tomita et al., 2005). While such global cloud resolving model (CRM) simulations currently represent the most natural way of simulating the interaction between deep cumulus convection and the large-scale circulation, they are extremely expensive computationally and can only be integrated for short periods of time even on state-of-the-art supercomputers. Shortcuts have been proposed to reduce computational costs, including the cloud resolving cumulus parameterization (CRCP or superparameterization) (Grabowski, 2001), where a small-domain CRM is used within each GCM grid box to replace the traditional cumulus parameterization, and the Diabatic Acceleration and REscaling (DARE) or equivalently the Reduced Acceleration in the VErtical (RAVE) approach (Kuang et al., 2005), hereafter KBB. Experiments with the CRCP approach show encouraging improvements over conventional GCMs in terms of, for example, the diurnal cycles of precipitation (Khairoutdinov et al., 2005). However, this approach does not simulate the convectively coupled waves well (Khairoutdinov et al., 2005) (their figure 14), a feature pointed out earlier by Grabowski (2001) and attributed to the artificial scale separation inherent to all parameterization approaches, including CRCP. Rather than impose an artificial scale separation, DARE reduces the computational cost by decreasing the scale separation.
between deep cumulus convection and the large-scale circulation. Recognizing the large scale separation between deep convection (a few kilometers and hours) and the large-scale circulation (thousands of kilometers and days), the DARE/RAVE approach assumes the interaction between the two can be largely preserved when the scale separation is reduced as long as it remains large. When the scale separation is reduced by a factor of $\gamma$, the saving in computational cost is $O(\gamma^3)$, bringing large-scale cloud-resolving simulations within reach of more modest computers.

Using equatorial $\beta$-plane simulations with the DARE/RAVE approach, KBB were able to produce convectively coupled equatorial waves with spectra and structures that compare well with observations. These results indicate that the DARE/RAVE approach and, more generally, coarse-resolution nonhydrostatic models (see Appendix), can be used to explicitly simulate convectively coupled waves.

In this paper, we present more complete results from equatorial $\beta$-plane simulations with the DARE/RAVE approach and compare them with observations. The model that we use and the experimental setup are described in section 2. In section 3, we present the results from two simulations, one with a single ITCZ (section 3a) and one with a double ITCZ (section 3b). The bias associated with the DARE/RAVE approach is assessed by repeating the double-ITCZ experiment with a smaller DARE factor (section 3c). We present our conclusions in section 4. An Appendix discusses the mathematical formulation of DARE, a derivation of its equivalence to RAVE, and some discussions of relevant earlier work.
2. Model and Experimental Setup

We use the System for Atmospheric Modeling (SAM) version 6.3, which is a new version of the Colorado State University Large Eddy Simulation / Cloud Resolving Model (Khairoutdinov and Randall, 2003). The model uses the anelastic equations of motion with bulk microphysics. The prognostic thermodynamic variables are the liquid/ice water static energy, total non-precipitating water and total precipitating water. The radiation schemes are those of the National Center for Atmospheric Research (NCAR) Community Climate Model (CCM3) (Kiehl et al., 1998). Readers are referred to Khairoutdinov and Randall (2003) for details about the model. For this study, we use a simple Smagorinsky-type scheme to represent the effect of subgrid-scale turbulence. The surface fluxes are computed using Monin-Obukhov similarity theory.

The experimental setup largely follows that of KBB. We use an effective horizontal resolution ($\gamma dx$) of 40 km with a DARE factor $\gamma=10$. The vertical grid spacing ranges from 125 m near the surface to 800 m in the upper troposphere, with the domain top at ~29 km. The domain has 576 grid points in the zonal direction with periodic boundary conditions, and 288 grid points in the meridional direction with solid walls at the boundaries, representing an equatorial $\beta$-plane that extends 5760 km on each side of the equator (~52°) and covers about half of the equatorial circle. We use perpetual equinox solar insolation and have removed the diurnal cycle by choosing a constant solar zenith angle for each latitude. The solar zenith angles are chosen to give the diurnally averaged solar insolation. The model fields are output twice per day. To reduce the cost of data analysis, we have horizontally block averaged all data into a 160 km by 160 km grid before processing. Given the large scale of the convectively coupled waves, this
reduction in the horizontal resolution has little effect on the results. Note also that the coarsened grid is still of a higher resolution than the 2.5-degree OLR data used in observational studies by, for example, Wheeler et al. (2000).

3. Simulated waves spectra and structures

In this section, we present the simulated wave spectra and structures and compare them with observations. Detailed descriptions of the observed waves can be found in, for example, Wheeler and Kiladis, (1999), Wheeler et al., (2000) and Straub and Kiladis, (2003), hereafter referred to as WK99, WKW00, and SK03.

a. A single-ITCZ case

We first discuss a case with a 20-meter mixed-layer ocean and zero oceanic heat flux, a setup similar to that of the single-ITCZ case shown in KBB. The results presented here are based on data from a 300-day period after the mixed-layer ocean has reached approximate equilibrium. Like KBB, the simulation has a single ITCZ on the equator (Fig 1b), although the mean SST (Fig. 1a) is significantly warmer (~3K) than that of KBB (their Fig. 2). This is largely caused by changes made to the microphysics scheme in the new version of the SAM model, which significantly reduces the shortwave cloud forcing, causing the increase in the equilibrium SST. This highlights the known sensitivity of the simulated climate by a CRM to its microphysics treatment (Grabowski, 2000). Simulations of the convectively coupled equatorial waves are, however, robust to these changes.

The simulated climatology of wind and temperature distributions, shown in Fig. 2, contains the basic features of the general circulation, subtropical jets in each hemisphere
and a mean meridional circulation with Hadley and (weak) Ferrel cells. While our idealized experimental setup precludes a direct comparison with observations, the simulated climatology is judged as sufficiently realistic for setting the background for the convectively coupled waves.

The zonal wavenumber-frequency spectra are computed based on the outgoing longwave radiation (OLR), largely following WK99. As our data record is only 300 days long, we have used a time window of 45 days instead of 96 days, with an overlap of 30 days between consecutive windows. The zonal extent of our domain gives a minimum zonal wavenumber of ~1.8.

The spectral analysis reveals a broadly red spatial-temporal spectrum (Fig. 3) with convectively coupled Kelvin waves appearing as the most prominent spectral feature (Figure 4). The Kelvin wave signal is so strong that it remains evident in the background spectrum, which has been smoothed with 10 passes of a 1-2-1 filter in wavenumber and frequency. As our simulation does not have a seasonal cycle, the statistics may be viewed as stationary in time. Therefore, uncertainties in the ratioed spectra (Fig. 4) are taken to be the standard deviation of the results from the 18 different temporal windows divided by square root of the estimated number of independent temporal windows (~6). Signals that do not exceed two times their uncertainties are shaded. While the number of independent estimates here are admittedly small, this calculation nonetheless gives a rough estimate of the statistical significance.

The dispersion relationship of these waves corresponds to that of a linear equatorial shallow water system with an equivalent depth of 12-90 m, as in observations (e.g., WK99). As convection is concentrated near the equator in this case, modes that require
deep convection off the equator are absent. These modes become prominent in the double-ITCZ case discussed in section 3b.

The construction of composite structures largely follows that of WKW00. For the Kelvin waves, the OLR averaged over 2.5N/S is filtered to contain only the Kelvin wave components (marked in Figure 4). We then regress the unfiltered data of each field against the filtered OLR data at a reference longitude. As our simulation is zonally symmetric, instead of choosing one particular reference point, we repeat the analysis with each longitude as the reference point, and then average the results with the reference points collocated. The standard deviation of the regression coefficients using different reference points is used as an estimate of the uncertainty (1-sigma). The basic features presented are all statistically significant (2-sigma).

The composite horizontal structures of wind and pressure at 12 km and the surface are shown in Fig.5. As in observations (e.g., WKW2000), the 12 km wind and pressure anomalies exhibit the classic linear equatorial shallow water Kelvin wave pattern: a positive pressure anomaly leading convection and a negative pressure anomaly following, and predominantly zonal wind anomalies. Using the relative magnitudes of zonal velocity ($U$) and pressure ($p$) anomalies, the classic equatorial wave theory gives a phase speed, $p/\rho U$, of $\sim20$ m s$^{-1}$, consistent with the actual phase speed of the waves seen in the spectra (Fig. 4). The amplitude of the zonal wind anomaly for the same OLR change is also comparable to that of WKW00 (their Fig. 5). Relative to their 12 km values, surface pressure and wind patterns are shifted eastward and of the opposite sign. Because of boundary layer friction, there is a stronger meridional wind component at the surface.
compared to the free troposphere, with meridional convergence leading convection and divergence lagging it.

The phase relationships between different heights are more clearly seen in the composite vertical structures, which are shown in Figure 6 for temperature (contours), zonal and vertical components of the winds (vectors), and moisture (shades) averaged between 2.5N/S. As in WKW00, the vertical wind component has been multiplied by a factor of 3000 to be consistent with the aspect ratio of the plot. The general patterns are similar to observations (compare with, e.g., Fig.7 in WKW00 and Figs. 3 and 5 in SK03). In the stratosphere, the signature of upward Kelvin wave propagation is visible in the temperature anomaly field. In regions of deep convection, the tropospheric temperature anomaly pattern shows a significant second baroclinic mode structure with warm temperature anomalies in the upper troposphere and cold temperature anomalies in the lower troposphere. The temperature and wind anomalies for the same change in OLR are comparable to those of WKW00 (their Fig. 7), but stronger than those in SK03. This may be explained by the fact that SK03’s vertical structures are based on data at 7.5°N: dynamical fields decay with distance from the equator as exp(-y²/R²), where y is the distance from the equator and R~12° is the equatorial deformation radius. We shall discuss this more in the next section with the double-ITCZ case, the analyses of which are done off the equator and thus more directly comparable with the results of SK03.

The general pattern of the moisture anomaly is also consistent with observations (e.g., in SK03). Before the onset of deep convection, there is a moist anomaly below ~2 km, and a dry anomaly in the lower and middle troposphere (above ~2km). In the region of upward motion, the lower and middle troposphere is moistened, accompanied by a drying of the
boundary layer. The maximum moisture anomaly in the middle troposphere is reached shortly after the maximum convective activity. A major difference with observations is that the observed moisture anomaly is stronger in the boundary layer than in the mid-troposphere, while in the simulations it is stronger in the mid-troposphere. While the simulated and observed moisture anomalies in the boundary layer are of comparable strength (for the same change in OLR), the simulated anomalies are much stronger in the mid-troposphere. Possible causes of this are discussed in section 3b.

The basic features of the simulated waves do not depend on the use of a mixed layer ocean or the exact location of the meridional boundaries provided they are sufficiently poleward. We have performed a simulation with the SST fixed to that of the equilibrium SST of the single-ITCZ case with a domain of 512 points in the zonal direction and 256 points in the meridional direction so that the boundaries are located at ~46N/S. The simulated waves are very similar to those presented in this section.

\textit{b. A double-ITCZ case}

We will now discuss a simulation with a 20 m mixed-layer ocean and an imposed oceanic heat flux, shown in Figure 7. The imposed oceanic heat flux is an idealization of the implied oceanic heat flux from an earlier fixed-SST run that produced a double ITCZ. Its main feature is the imposed oceanic cooling in the tropics, which peaks at 40 W m$^{-2}$ on the equator. At high latitudes, the heat flux is chosen to maintain the equilibrium SST close to the initial SST distribution and is not important to the simulation of the waves. The results presented below are based on data from a 300-day period after the SST has reached approximate equilibrium.
In this case, a double-ITCZ forms at 7.5N/S (Figure 8). The resulting spectra (Figure 9) capture the main wave types seen in observations (WK99). With convection off the equator, modes such as the n=0 mixed Rossby-gravity (MRG) waves appear (Figure 9), although the Kelvin wave signal remains the strongest. A similar pattern is found in observations. Following WK99, we further separate the n=0 MRG waves into an n=0 eastward inertial gravity (EIG), and an n=0, zero zonal wavenumber inertial gravity wave, and use MRG only for the westward propagating ones.

The simulated spectra have relatively weak signals in the equatorial Rossby (ER) wave region. Previous studies have suggested that the ER waves are sensitive to background shear (Wang and Xie, 1996). Because of the idealized setting, the background state of the simulation does not fully represent that in the real atmosphere. This may have contributed to the weakness of the simulated ER waves.

Although the time-mean precipitation (Fig. 8b) shows two ITCZs, there is an interesting low frequency (~1/20 days) signal at zero zonal wavenumber in the antisymmetric spectrum, corresponding to a competing ITCZ mode in which the stronger branch of the ITCZ alternates between north and south of the equator. A competing ITCZ has been noted in observational studies of the eastern pacific during boreal spring when a double-ITCZ forms (Gu et al., 2005). Processes involved in producing this mode will be discussed in a separate paper. Finally, there is no MJO signal in this simulation. The reason for this is not yet clear and is being actively investigated.

The composite structures of the Kelvin waves are constructed in the same way as in section 2a, except with OLR and other variables averaged over 5-10N and 5-10S because the convective signals are located off the equator. The composite structures are in general
similar to those of the single ITCZ case, despite some differences that are related to the
fact that deep convection is located off the equator.

Because the vertical composites are now formed with averages 5-10 degrees off the
equator, the results can be directly comparable to those of SK03. The amplitude of
temperature and zonal wind anomalies in the upper troposphere for the same OLR change
is comparable to that from SK03’s analysis of the radiosonde data (their Fig.5), but is
greater than that from reanalysis data (their Fig. 3). Temperature anomalies in the lower
troposphere, on the other hand, are significantly stronger in the simulation than in the
observations. Again, while the simulated and observed moisture anomalies in the
boundary layer are of comparable strength, the simulated anomalies are much stronger in
the mid-troposphere. The exact cause of these discrepancies warrants further
investigation. We suspect that the exaggerated temperature and moisture anomalies may
be associated with the same issue. In particular, we speculate that they may be related to
the coarse resolution nature of our simulation, which has an effective horizontal
resolution of 40 km. For instance, a reduced level of lateral entrainment by cumulus
updrafts, and hence a reduction in their sensitivity to mid-troposphere moisture, might
give rise to the distortion seen in the simulated waves.

The composite structures for the MRG and EIG waves are shown in Figs 12-15. These
composites are constructed in a similar way to those of the Kelvin wave, except that
differences between the 5-10N and 5-10S averages are used for the regression. We have
included the zero wavenumber n=0 inertial gravity wave in the construction of the EIG
composites.
The horizontal structures of the simulated MRG and EIG waves resemble the observed patterns (WKW00). Besides the good resemblance to classic equatorial wave patterns, the simulation also captures well the observed tilt of the OLR anomalies (northeast to southwest in the northern hemisphere) of the MRG waves.

The vertical composite structure of the MRG and EIG waves are also similar to those of WKW00 in terms of the temperature and velocity fields. Compared to observations, the simulated waves have stronger eastward propagating components than the zero wavenumber component (Fig. 9). Hence, the simulated EIG waves have a more pronounced eastward propagation than observations. The composite structures for moisture are not yet available, although the simulated moisture fields for MRG and EIG also have stronger anomalies in the mid-troposphere than in the boundary layer, similar to the simulated Kelvin waves.

Again, basic features of the simulated waves do not depend on the inclusion of a mixed layer ocean or the exact SST distribution, as confirmed by a simulation where the SST is prescribed to be

\[
\text{SST}(K) = 301.5 + 20(4b - c - 3)/3
\]

\[
b = \frac{1}{2} \left(1 + \cos \left(\frac{2\pi y}{8000\text{km}}\right)\right)
\]

\[
c = \frac{1}{2} \left(1 + \cos \left(\frac{\pi y}{8000\text{km}}\right)\right)
\]

where \(y\) is the distance from the equator. The functional form is borrowed from (Raymond, 2001). In this case, a double-ITCZ forms even though the SST distribution is essentially flat near the equator (Figure 16). This is mainly because of the stronger
surface winds off the equator which drive stronger surface fluxes, thus favoring convection off the equator.

The spectra of this case (Figure 17) are in general similar to the mixed layer ocean case, so are the composite wave structures (not shown). However, antisymmetric modes do appear to have some sensitivity to conditions at the extratropics. Additional experiments suggest that they also have some sensitivity to the location of the meridional boundaries.

c. Sensitivity to DARE/RAVE

To evaluate the sensitivity of our results to DARE/RAVE, we have repeated the fixed SST experiment described in section 3b with an effective horizontal resolution of 16 km and $\gamma=4$. The zonal extent of the domain is halved to reduce the computational cost. The SST is prescribed to be that of Fig 16a, and the time averaged zonal mean precipitation is shown in Fig 16b. The ITCZ appears slightly broader with the higher horizontal resolution, a behavior noted previously in the global cloud resolving simulations of Tomita et al. (2005). It also moves slightly poleward compared to the coarser resolution case. Figure 18 shows the spectra based on the last 250 days of a 300 day simulations.

Because of the smaller zonal extent of the domain, the spectra of the fine resolution simulation are more quantized in zonal wavenumber (lowest zonal wavenumber is ~4). The basic spectral features are nonetheless clear and similar to those of the simulation with a coarser resolution (i.e., a greater DARE factor $\gamma$). Structures of the simulated waves are also similar. Here we only show that for the convectively coupled Kelvin waves. The similarity between the results with $\gamma=10$ and $\gamma=4$ should help alleviate concerns that the simulated waves are artifacts due to the DARE/RAVE approach. The
higher horizontal resolution simulation still exaggerates the temperature and moisture anomalies in the lower and middle troposphere, presumably because the effective horizontal resolution is still rather coarse (16km). We emphasize, however, that the exact cause of this distortion is not well understood and warrants further investigation.

4. Conclusions

The main goal of the present paper is to demonstrate that numerical simulations using the DARE/RAVE approach proposed by KBB can capture the basic features of the convectively coupled equatorial waves and can therefore be a tool for further studying such waves. Toward this goal, we have presented results from equatorial β-plane experiments using the DARE/RAVE approach. Despite some differences noted in this paper, the simulations reproduce well the structure and spectra of the observed waves. As discussed in the Appendix, the DARE/RAVE approach is in essence one formulation of coarse resolution nonhydrostatic models, and therefore is expected to introduce errors compared with global cloud-resolving simulations such as those reported in Tomita et al. (2005). However, our sensitivity test indicates that basic features of the simulated waves remain unchanged when the equivalent resolution is changed from 40km to 16km. This, combined with the similarity between the simulated and observed waves, leads us to conclude that our simulations of the convectively coupled equatorial waves are not artifacts of our simulation approach. Because of the significant reduction in computational costs as compared to global cloud-resolving models, we believe that simulations like those reported here are useful for studying the convectively coupled equatorial waves. In particular, they can be used to obtain data that are difficult to observe, such as convective heating and momentum transfer, and also to carry out
perturbation experiments where various components are removed to evaluate their impact. Results from such studies will be presented in a separate paper.
The Diabatic Acceleration and Rescaling (DARE) approach was introduced in KBB along with its motivations and its equivalence to the Reduced Acceleration in the Vertical (RAVE) interpretation. While the presentation in KBB was clear, from the feedback that we have received, it appears that it is of interest to also present the approach in a more mathematical manner and to show its equivalence to the RAVE. This motivates the description in this appendix.

Consider the equations of motion in Cartesian coordinate in a rotating frame

\[
\begin{align*}
\frac{dV}{dt} &= -\rho^{-1} \nabla p - 2\Omega \cdot V + 2\Omega_H \cdot w + F_H \\
\frac{dw}{dt} &= -\rho^{-1} \frac{\partial p}{\partial z} - g + 2\Omega \cdot V + F_z \\
\frac{d\rho}{dt} + \rho \left( \nabla \cdot V + \frac{\partial w}{\partial z} \right) &= 0 \\
\frac{d\Phi}{dt} &= P_\phi + F_\phi
\end{align*}
\]

(1)

where \( V = (u, v) \) is the horizontal velocity, \( w \) is the vertical velocity,

\[
\frac{d}{dt} = \frac{\partial}{\partial t} + V \cdot \nabla + w \frac{\partial}{\partial z}
\]

is the Lagrangian derivative, \( \rho \) is density, \( p \) is pressure, \( g \) is the combined acceleration by gravity, and the centrifugal force, \( \Omega_H = (\Omega_x, \Omega_y) \), where \( \Omega_{x,y,z} \) are the three components of the rotation vector. We have defined \( V^\perp = (-v, u); \Omega_H^\perp = (-\Omega_y, \Omega_x) \). The vector \( \Phi \) represents the prognostic thermodynamic variables. In the SAM model, these are liquid/ice water static energy, total nonprecipitating water, and total precipitating water. In other models, they could be, for example, potential temperature and concentrations of water vapor, cloud ice, water,
rain, snow, graupel and/or other hydrometeors. The variable $P_\Phi$ denotes the corresponding production terms. When $\Phi$ is the liquid/ice water static energy, $P_\Phi$ denotes radiative heating and sources from the precipitation process. When $\Phi$ is the concentration of a water/hydrometeor species, $P_\Phi$ denotes the source term due to microphysical processes. $F_\Phi, F_H=(F_x, F_y)$ and $F_z$ are the sources of $\Phi$, horizontal and vertical accelerations due to subgrid scale processes, respectively.

The system is closed by including the equation of state, and equations for $F_{H,z,\Phi}$ (subgrid scale model), $P_\Phi$ (radiative transfer and microphysics parameterizations) and $S_{V,\Phi}$ (surface layer model; radiation boundary condition is assumed for the upper boundary). While the actual parameterizations can be very complicated and model dependent, they can be written in general forms to indicate their dependence on other variables as the following:

\[
\rho = \rho(p, \Phi) \\
F_{H,z,\Phi} = \mathbb{F}_{H,z,\Phi} \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, V, w, \Phi \right) \\
P_\Phi = \mathbb{P}_\Phi \left( \frac{x}{x_0}, \frac{y}{y_0}, \frac{t}{t_0}, z, p, \Phi \right) \\
S_{V,\Phi} = \mathbb{S}_{V,\Phi} \left( V(z_{ref}), \Phi(z_{ref}), \Phi_{sfc}, z_{ref} \right)
\]

This is appropriate as explained presently. First, the equation of state only involves $p$, $\rho$ and $\Phi$. Note that this already includes the effect of water vapor and hydrometeors.

Second, generally speaking, current bulk microphysics parameterizations, such as the one used in the SAM model, treat conversion and production rates of various hydrometeors as functions of thermodynamic variables and pressure only so that $P_\Phi = \mathbb{P}_\Phi(p, \Phi)$. Radiative transfer calculations have additional space and time dependences, but dependences on the...
horizontal coordinate and time can be conveniently written in a nondimensional form (i.e. latitude, longitude, and phase in the seasonal/diurnal cycle). If spherical geometry is used in the radiative transfer calculation, an additional dependence on the aspect ratio needs to be included. However, this is not necessary for climate models and CRMs where the plane parallel approximation is sufficient. This gives the third expression in Eq (2). The surface fluxes here are written as general functions of the horizontal winds and thermodynamic conditions at a reference height \( z_{ref} \), the reference height itself, and the surface conditions \( \Phi_{sfc} \), which include surface temperature, humidity, surface roughness etc. The roughness length over a water surface can be estimated from \( V(z_{ref}), \Phi(z_{ref}), z_{ref} \), and \( \Phi_{sfc} \).

The model subgrid scale parameterization is written as general functions of the velocity and scalar fields and their spatial gradients. As an illustrative example, consider the vertical acceleration due to subgrid scale processes based on a first order eddy viscosity model

\[
F_z = \rho^{-1} \left[ \nabla \cdot \left( \rho \nu \left( \nabla w + \frac{\partial \mathbf{V}}{\partial z} \right) \right) + \frac{\partial}{\partial z} \left[ 2 \rho \nu \frac{\partial w}{\partial z} \right] \right] \tag{3}
\]

Here we see the dependence of \( F_z \) on the velocity gradient from the rate-of-strain tensor. The eddy viscosity \( \nu \) can also be written formally as a function of the velocity and scalar fields and their spatial gradients. In the Smargorinsky model, for example, the eddy viscosity is related to the filtered rate of strain, while in a turbulent kinetic energy (TKE) closure, the eddy viscosity is related to the TKE, which is further related to the velocity and scalar fields and their spatial gradients through the TKE equation. In this latter case,
the \( F \) functions should be viewed in a general sense to include the integration of the TKE equation.

In the DARE approach, we rescale the following variables

\[
[\hat{x}, \hat{y}, \hat{t}] = \gamma^{-1}[x, y, t]
\]

\[
[\hat{\Omega}_z, \hat{\omega}, \hat{P}_\phi, \hat{S}_{\phi, f}] = \gamma[\Omega_z, \omega, P_\phi, S_{\phi, f}]
\]

Variables in the DARE approach are shown with a hat. We keep all equations / parameterizations and all other variables unchanged. The equations used in the DARE approach are therefore

\[
\frac{D\hat{V}}{Dt} = -\hat{\rho}^{-1}\hat{V}\hat{\rho} - 2\hat{\Omega}_z\hat{V} + 2\hat{\Omega}_z\hat{\omega} + \hat{F}_\mu
\]

\[
\frac{D\hat{\omega}}{Dt} = -\hat{\rho}^{-1}\frac{\partial \hat{\rho}}{\partial \hat{z}} - \hat{g} + 2\hat{\Omega}_\mu \cdot \hat{V} + \hat{F}_z
\]

\[
\frac{D\hat{\rho}}{Dt} + \hat{\rho}\left(\hat{V} \cdot \hat{V} + \frac{\partial \hat{\omega}}{\partial \hat{z}}\right) = 0
\]

\[
\frac{D\hat{\Phi}}{Dt} = \hat{P}_\phi + \hat{F}_\phi
\]

where \( \frac{D}{Dt} = \frac{\partial}{\partial \hat{t}} + \hat{V} \cdot \hat{V} + \hat{\omega} \frac{\partial}{\partial \hat{z}} \), and

\[
\hat{\rho} = \hat{\rho}(\hat{\rho}, \hat{\Phi})
\]

\[
\hat{F}_{H,z,\Phi} = \mathbb{F}_{H,z,\Phi}\left(\frac{\partial}{\partial \hat{x}}, \frac{\partial}{\partial \hat{y}}, \frac{\partial}{\partial \hat{z}}, \hat{V}, \hat{\omega}, \hat{\Phi}\right)
\]

\[
\hat{P}_\phi = \gamma \mathbb{P}_\phi\left(\hat{x}, \hat{y}, \hat{t}, \hat{z}, \hat{\rho}, \hat{\Phi}\right)
\]

\[
\hat{S}_{V,\phi} = \gamma \mathbb{S}_{V,\phi}\left(\hat{V}(\hat{z}_{ref}), \hat{\Phi}(\hat{z}_{ref}), \hat{\Phi}_{ref}, \hat{z}_{ref}\right)
\]
Note the functional forms of \( R, F_{H,z,P}, S_{V,\Phi} \) (i.e., the parameterizations) are not changed. If we replace all rescaled variables (with hat) by variables without rescaling in Eq. (5) and (6), we have

\[
\begin{align*}
\frac{d\mathbf{V}}{dt} &= -\rho^{-1}\nabla p - 2\Omega_z \mathbf{V}^\perp + 2\Omega_{zH}^\perp \mathbf{w} + \mathbf{F}_H \\
\gamma^2 \frac{dw}{dt} &= -\rho^{-1} \frac{\partial p}{\partial z} - g + 2\Omega_{zH}^\perp \mathbf{V}^\perp + F_z \\
\frac{d\rho}{dt} + \rho \left( \nabla \cdot \mathbf{V} + \frac{\partial w}{\partial z} \right) &= 0 \\
\frac{d\Phi}{dt} &= P_\Phi + F_\Phi
\end{align*}
\]

where

\[
\begin{align*}
\rho &= \rho(p, \Phi) \\
F_{H,\Phi} &= \gamma^{-1} F_{H,\Phi} \left( \gamma \frac{\partial}{\partial x}, \gamma \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \mathbf{V}, \gamma w, \Phi \right) \\
F_z &= F_z \left( \gamma \frac{\partial}{\partial x}, \gamma \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \mathbf{V}, \gamma w, \Phi \right) \\
P_\Phi &= P_\Phi \left( x, y, \frac{t}{t_0}, z, p, \Phi \right) \\
S_{V,\Phi} &= S_{V,\Phi} \left( \mathbf{V}(z_{ref}), \Phi(z_{ref}), \Phi_{sfc}, z_{ref} \right)
\end{align*}
\]

Here, all variables are not rescaled, but the equations are modified as compared to Eq. (1) and (2). This is the RAVE interpretation. It is of course equivalent to DARE, i.e., Eq. (5) and (6), given Eq. (4). Note that in Eq. (8), the equations for \( F_{H,z,\Phi} \), i.e., the subgrid scale parameterizations, are modified, because of the presence of \( \gamma \) inside of the \( F_{H,z,\Phi} \) functions. Using vertical acceleration from a first-order eddy viscosity model again as an illustrative example, the subgrid scale model appropriate for RAVE is
Changes in the other subgrid scale terms and the eddy viscosity can be done in a similar manner.

The DARE interpretation was discussed in some detail in KBB, and the RAVE interpretation provides additional insights into the nature of this approach. RAVE makes it clear that dynamics of the large-scale circulation, which is approximately hydrostatic and inviscid, is preserved in this approach, and the effects of the modifications are only felt directly on horizontal scales small enough for non-hydrostatic effects and subgrid-scale turbulence to become important. It also makes it clear that the present approach is one type of coarse resolution non-hydrostatic model: a DARE simulation of a horizontal grid size of $\Delta x$, $\Delta y$ and a DARE factor $\gamma$ is equivalent to a simulation of a horizontal grid of $\gamma\Delta x$, $\gamma\Delta y$ using equations (7) and (8) without rescaling the variables. For the experiments presented in KBB and in sections 3a and 3b of the present paper, $\Delta x=\Delta y=4\text{km}$ and $\gamma=10$, so the effective horizontal resolution is 40km. The DARE-RAVE equivalence shows that this coarse resolution non-hydrostatic model is formulated in a way that its dynamical behavior at small scale resembles that of a fine resolution model\footnote{One could also understand the effect of RAVE as changing the sound and gravity wave dispersion relationships, or alternatively, as increasing the non-hydrostatic effects of the system (recall that $\gamma=0$ corresponds to the hydrostatic approximation, $\gamma=1$ corresponds to an unmodified system, and $\gamma>1$ in RAVE). However, the more direct way of seeing its behavior on convective scales, in our opinion, is through its equivalence to DARE.}.

For convective scale motions without rotation and radiative and surface forcing, the behavior of a RAVE system is the same as that of a DARE system with a narrower horizontal scale (i.e. a smaller aspect ratio) described by the standard equations except...
with accelerated microphysics. While it does distort the convective dynamics, the accelerated microphysics in the DARE interpretation was chosen to preserve the large-scale behavior, such as the stratiform cloud/rain, which has a major impact on the radiative balance. Within the DARE framework, it is possible to accelerate microphysics only for the large-scale/stratiform processes. The performance of such a formulation warrants further investigation.

The RAVE interpretation, without changes to the subgrid scale terms, has the same formulation as what was proposed by Browning and Kreiss (1986) hereafter BK86, based on a more general theory on hyperbolic systems with different time scales (Kreiss, 1980). This was brought to our attention after the publication of KBB. BK86 proposed Eq. (7) in the context of removing the ill-posedness of open boundary value problem for primitive equations. By slowing down gravity and sound waves (straightforward to show or to infer by its equivalence to DARE) instead of removing them altogether, Eq. (7) allows the system to retain its hyperbolicity so that the open boundary value problem is well posed. The reader is referred to that paper and their later publications for more discussions.

More recently, the approach of BK86 has been applied to a full physics mesoscale weather prediction model, named a quasi-nonhydrostatic model (QNH), and was found to be useful (MacDonald et al., 2000). Interestingly, the same formulation (Eq. 8) also arose in the Lattice-Boltzmann approach to simulating ocean circulation (Salmon, 1999). However, the utility of this approach (combined with the subgrid scale treatment) in simulating the interaction between large-scale circulation and deep cumulus convection was, to the authors’ knowledge, first proposed by KBB.
The results presented in this paper and in KBB serve as empirical evidence that the DARE/RAVE approach, and more generally, coarse resolution non-hydrostatic models can indeed be good tools for simulating the convectively coupled equatorial waves. As DARE/RAVE is only one type of coarse resolution non-hydrostatic models, alternative formulations and the utility of coarse resolution non-hydrostatic models in other problems involving the interaction between deep cumulus convection and large-scale circulation warrant further investigation. In particular, the performance of the DARE/RAVE approach combined with a shallow cumulus scheme is currently being examined.
Acknowledgements

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Figure Captions:

Figure 1 Climatological zonal mean SST (a) and precipitation (b) for the single ITCZ case.

Figure 2 Climatological zonal mean (a) zonal wind (black contours; contour interval 5 m/s) and temperature (gray contours; contour interval of 5 K). The heavy contour shows the zero wind line and negative contours are dashed (b) meridional and vertical winds. The length of the component has been amplified to be consistent with the aspect ratio of the plot. The maximum meridional and vertical velocities are approximately 2.6 and 0.013 m s⁻¹, respectively.

Figure 3 Zonal wavenumber-frequency spectrum of the base-10 logarithm (with an arbitrary offset) of the background power of the OLR field of the single ITCZ case.

Figure 4 Symmetric OLR power divided by the background spectrum shown in Fig. 3. Contour interval is 1.0. The heavy lines demarcate the spectral region used to filter Kelvin waves. The dashed lines show the dispersion relation for Kelvin waves with equivalent depths of 8, 12, 25, 50, and 90 m. Signals that do not exceed two times their uncertainties are shaded.

Figure 5 Zonal anomalies of (a) 12 km zonal and meridional wind (arrows; maximum absolute zonal and meridional velocities are 1.7 and 0.4 m s⁻¹, respectively), pressure (contours; interval of 4 Pa), and negative values of OLR (shading; interval of 3 W m²) (b) surface zonal and meridional wind (arrows; maximum absolute zonal and meridional velocities are 0.8 and 0.2 m s⁻¹, respectively), pressure (contours; interval of 15Pa) and precipitation (shading; interval of 1 mm/day). In all panels, dashed contours and light shading indicate negative values and solid contours and dark shading indicate positive values.

Figure 6 Average from 2.5S to 2.5N of anomalous (top panel) OLR and (bottom panel) zonal and vertical wind (arrows; maximum absolute values for zonal and vertical wind are approximately 3.0 and 0.008 m s⁻¹, respectively), temperature (contours; interval of 0.1 K), and water vapor (shading; interval of 0.05 g kg⁻¹). Light shading and dashed contours indicate negative values; dark shading and solid contours indicate positive values.

Figure 7 Imposed oceanic heat flux in the double-ITCZ case.

Figure 8 Climatological zonal mean SST (a) and precipitation (b) for the double ITCZ case.

Figure 9 (a) asymmetric and (b) symmetric OLR power divided by the background spectrum. Dashed lines show (a) MRG and (b) Kelvin dispersion relations with equivalent depths of 8, 12, 25, 50, and 90 m. Thick lines demarcate spectral regions used to filter MRG, EIG, and Kelvin modes.
Figure 10  As in Fig. 5 but for the double ITCZ simulation. Contour intervals and arrow lengths are the same as in Fig. 5. Maximum absolute surface zonal and meridional velocities are 1.4 and 0.25 m s\(^{-1}\), respectively; maximum 12 km zonal and meridional velocities are 2.6 and 0.5 m s\(^{-1}\). Regions in which the data are not significant at 2 sigma are blocked out.

Figure 11  As in Fig. 6 but for double ITCZ simulation. Contour intervals and arrow lengths are the same as in Fig. 6. Maximum absolute zonal and vertical velocities are 2.1 and 0.06 m s\(^{-1}\), respectively.

Figure 12  As in Fig. 10, but for MRG mode. Contour intervals are (a) 1 Pa and 1.5 W m\(^{-2}\), (b) 5 Pa and 0.5 mm day\(^{-1}\). Unit arrows are twice as long as those in Fig. 10. Maximum absolute surface zonal and meridional wind velocities are 0.5 and 0.6 m s\(^{-1}\); maximum 12 km zonal and meridional wind velocities are 0.4 and 1.2 m s\(^{-1}\). Regions that are not significant to 2 sigma are blocked out.

Figure 13  As in Fig. 10, but for EIG mode. Arrow lengths and contour intervals are the same as in Fig. 10. Maximum absolute surface zonal and meridional wind velocities are 0.5 and 0.5 m s\(^{-1}\); maximum 12 km zonal and meridional wind velocities are 0.5 and 0.9 m s\(^{-1}\).

Figure 14  As in Fig. 11, but for the MRG mode. Contour intervals are half those in Fig. 11; unit arrow lengths are twice those in Fig. 11. Maximum zonal and vertical wind velocities are 1.4 and 0.008 m s\(^{-1}\), respectively.

Figure 15  Same as Fig. 11, but for EIG mode. Contour intervals and arrow lengths are the same as Fig. 11. Maximum absolute zonal and vertical velocities of 1.6 and 0.01 m s\(^{-1}\), respectively.

Figure 16  The prescribed SST distribution (a) and the equilibrium zonal mean precipitation distribution (b) for the fixed SST case in section 3b (solid) and the finer resolution case in section 3c (dashed).

Figure 17  Same as Fig. 9 but for the fixed SST case.

Figure 18  Same as Fig. 9 but for the case with an effective horizontal resolution of 16km.

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