Describing Spin-2 Bose-Einstein Condensation in Optical Lattice With Topological defects

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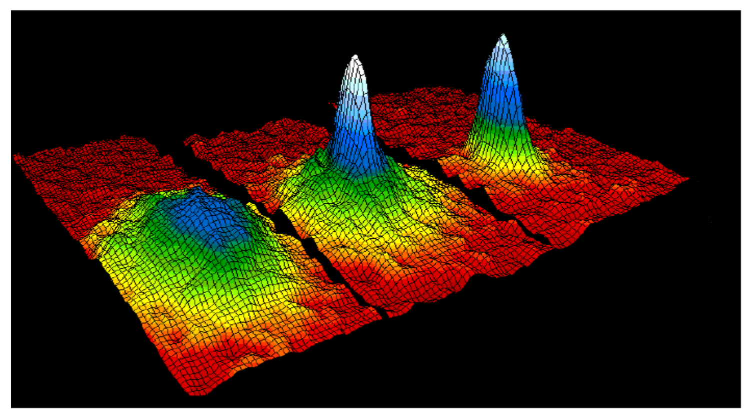
Introduction:

Bose-Einstein Condensation is a new form of matter that was originally predicted by Einstein in 1925 but could not be realized until 1995 by a group led by Eric Cornel and Carl Wieman. BEC can only be realized at low temperature (it was first realized at 170nK), which made it difficult to produce BEC.



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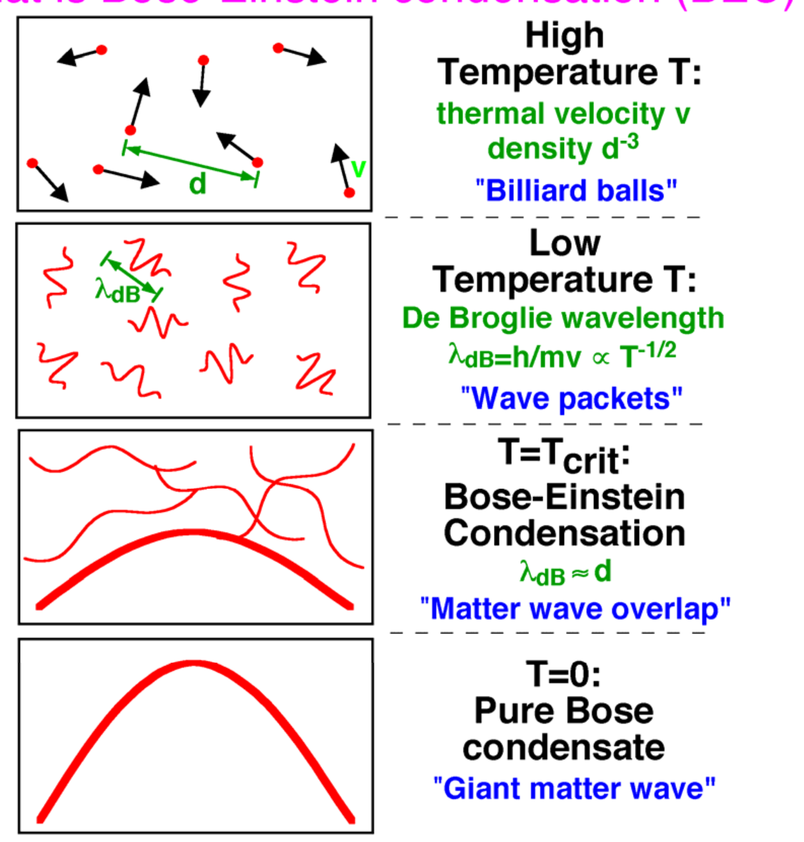
What is Bose-Einstein condensation (BEC)?



From Kettetle's website @ MIT

Objective:

Determination of the ground state (lowest energy state) of spin-2 BEC in optical lattice. Since particles in nature seeks the lowest energy state, it is the ground state that people realize in experiments. Therefore, theorists can confirm their theories if they could predict measurements of observables in the ground state. In particular, we are interested in classifying topological defects in this system, which can be detected in experiments. The observation of topological defects tells us order-parameter space of the system, through which we hope to confirm the correctness of our model.

Difficulty

1.Many Body System: Constructing a model (Hamiltonian) and finding the energy of the system becomes increasingly more difficult as the number of particles in the system increases. Here, we are dealing with the system with 10^12 particles.

2. Quantum Mechanical System: BEC is inherently quantum mechanical object. In optical lattice, particles have spin-freedom, so we have to take into account spin-interactions when we compute the energy of a state.

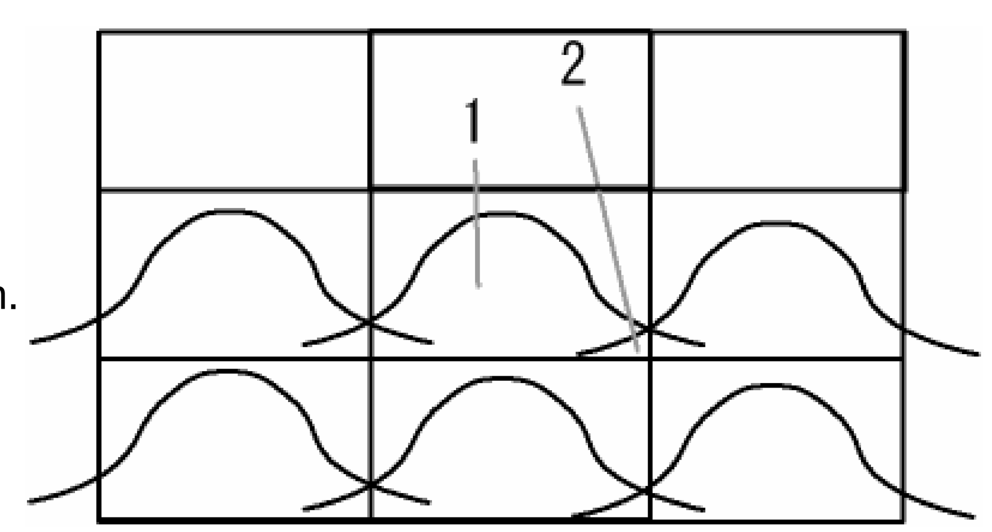
Model:

$$H = \frac{1}{2}U_0 n(n-1) + \frac{1}{2}U_1 P_0 + \frac{1}{2}U_2(F^2 - 6n)$$
$$-J \sum_{<ij>} (a^\dagger_{i\alpha} a_{j\alpha} + a^\dagger_{j\alpha} a_{i\alpha})$$

Because of the difficulty noted above, we want to make the model of this system as simple as possible. We consider two effects in constructing a model (Hamiltonian)

1. On-site interaction between particles (First line, main interaction)
2. Interaction between particles in different sites (Second line, perturbation)

We assume that J is small compare to other interaction parameter, U. Then we can treat the interaction between particles in different sites as perturbation.



Method:

This Hamiltonian is still too hard to deal with to find the ground state of the system. In order to find the ground state of the system as well as to find the symmetry of the ground state, we resort to the following methods.

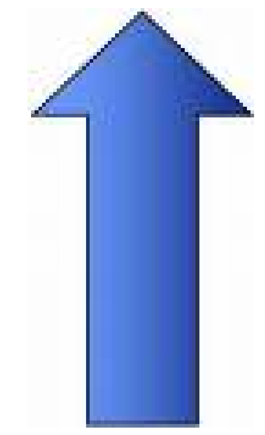
Perturbation Theory: We simplify the Hamiltonian by replacing J term with effective Hamiltonian

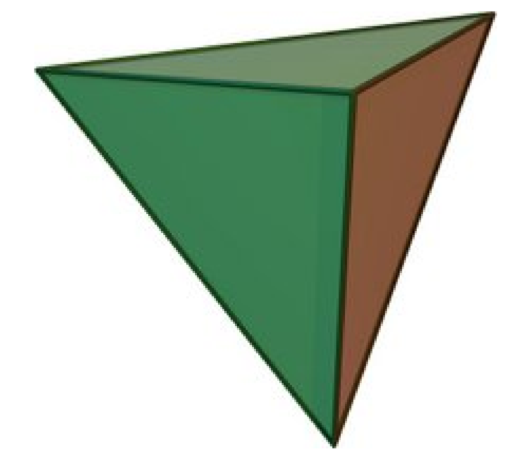
Variational Method: We find the ground state that minimizes the (effective) Hamiltonian

Classification Method (R.Barnet, A. Turner, E. Demler): We find the symmetry of the ground state

Result:

Each shape represent the symmetry group of the state. Each state represents the ground state of different interaction strength. From the left, we call each state as "ferromagnetic state," "tetrahedroratic state," and "nematic state."





$$|\psi_A >= |2>$$

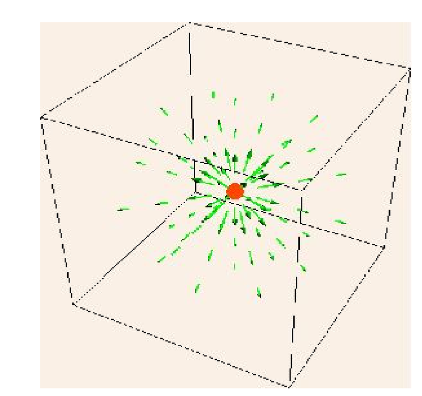
$$|\psi_{B1} >= \frac{1}{2}|-2> + \frac{1}{\sqrt{2}}|0> - \frac{1}{2}|2>$$
$$|\psi_{B2} >= \sqrt{\frac{2}{3}}|-1> + \sqrt{\frac{1}{3}}|2>$$

$$|\psi_C >= \frac{1}{\sqrt{2}}\sin(\eta)|-2> + \cos(\eta)|0> + \frac{1}{\sqrt{2}}\sin(\eta)|2>$$

Topological Defects:

Topological defects are the "excited" state of the system, whose excitation is caused by the topology of underlying order-parameter space. Once a topological defect is created, system cannot fall down to a ground state because topology doesn't allow continuous transformation from topological defects to ground states. By considering a continuous change of spin in space, and mapping the spin directions into order-parameter space, we find topological defect corresponds to non-trivial homotopy groups.

Computation of homotopy group is made easy by the following formulae, where G is the universal cover of order-parameter space with group structure, H is the lift of symmetry group of the ground state, and H_{0} is the path-connected space containing the identity.



$$\pi(G/H) = H/H_0$$

Then we find, for example, that ferromagnetic state (A), has $\pi(G/H) = Z_4$ whereas nematic state has $\pi(G/H) = Z \times Z$ and tetrahedratic state (B) has 8 distinct topological defects.

Conclusion:

With some understanding of ground state of spin-2 BEC, one can go on to study quantum phase transition of spin-2 system. Some researchers suggest the connection between high-temperature superconductivity and quantum phase transition, therefore, both the theory and application of this field remains exciting in the future .

Acknowledgement:

I learned the works described here through numerous papers. I read papers by A.Imambekov, E.Demler, T. Ho, A.Turner, and R.Barnett most thoroughly. I thank Ari Turner, Ryan Barnet and Adilet Imambekov for many useful discussions. Finally, I thank Eugene for having me for the summer.