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Lecture 15

15 Suppression of Decoherence

Errors in quantum states due to the decoherence of qubits can be suppressed by using specific algorithms (error correction) or by using physical control. Here we will discuss the second approach. There are several common methods. They are often combined together to provide better results.

15.1 Design of a qubit I

One finds systems which are isolated from their environments, but still controllable by external fields. For example, nuclear spins $I = 1/2$ have very weak coupling to phonons. The complication in this case is the control.

Another example can be a quantum system with a large energy gap (superconductor). If the gap is large and the energy spectrum of a bath has a high frequency cut-off, then the system cannot absorb (here we talk about real processes) single quanta of the bath excitation. Addiontally, the system can relax from the excited state by emitting several quanta of the bath excitation. If the coupling is weak then the probability of such processes is small.

Virtual processes are less suppressed.

15.2 Control of environment

This is the most trivial approach. If we are talking about decoherence due to the coupling to phonons, then the easiest way to control the environment is to put the system into a cryostat. If we want to suppress the coupling of a spin qubit to a bath of other spins we can try to use isotopically engineered materials, etc. The limitations of this approach are:

- the lower we go in temperature the harder it is to keep the system in thermal equilibrium,
- some thermal processes you cannot suppress by lowering temperature, for example, spontaneous emission,
- external qubit control always disturbs the environment,
- for a spin bath isotope purification does not always work, for example within III-V semiconductors, the natural distribution of all the elements have non-zero nuclear spins,
- even if the material can be isotopically purified ,for example Si can be purified to $c(^{29}Si) \approx 0.01\%$, one cannot control precisely microscopic environment of each qubit.

15.3 Design of a qubit II. Decoherence free subspace

One can select a subsystems where particular types of errors are suppressed. For example, a subspace of two qubits:

$$|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle - i|1\rangle|0\rangle), \tag{1}$$

$$|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle + i|1\rangle|0\rangle). \tag{2}$$

If we have any phase error that affects both qubits in the same way as

$$|1\rangle \to e^{i\phi}|1\rangle, \tag{3}$$

$$|0\rangle \to |0\rangle, \tag{4}$$

then, due to error, both of the states will get the same phase. But quantum states are defined up to a phase, therefore there is no error.

15.4 Dynamical decoupling I. Bang-bang control

This method is adopted from NMR. In NMR absorption lines in liquids are much narrower than in solids. There is, so called, dynamical averaging. If spins interacting with each other move randomly then the interaction between them is averaged. Similarly in solids, to suppress interaction of a single spin to the environment one can flip this spin. Then at some stroboscopically selected times the effect of the environment will be partly compensated. We have seen this in the example of spin echo. However in this case, there is no single qubit decoherence.

As another example, we can consider spin echo with spectral diffusion [1]. The magnetic π-pulse not only removes inhomogeneous broadening but also suppresses spectral diffusion. For one more example see Ref. [2].

15.5 Dynamical decoupling II

When we know the particular interaction that we want to eliminate, we may use this knowledge in the design of our control scheme, as in the following example.

15.6 Example: Magic echo

There is a system of interacting spins. In CaF_2, studied in Ref. [3], it is a system of fluorine nuclear spins $I = 1/2$. Each spin interacts with all the others by dipolar interaction. If at time $t = 0$ we initialize all the spins in a factorizable superposition state $\prod_i(|\uparrow_i\rangle + |\downarrow_i\rangle)$, then it evolves into a general (non-factorizable) state due to the dipole-dipole interaction between spins. Every spin will undergo a loss of coherence. However, this decoherence process can be inverted by an ingeniously designed control scheme.

Let us consider the dynamics of a nuclear spin pair. We can solve this problem exactly. The echo can be evaluated as

$$\text{Echo} = \text{Tr}\{(S_1^y + S_2^y)U(t_2)e^{i\pi/2(S_1^y+S_2^y)}B_x(t_b)e^{-i\pi/2(S_1^y+S_2^y)}U(t_1)\rho_{0+} \times \text{h.c.}\}, \tag{5}$$

where

$$U(t) = e^{-iH_{dd}t}, \tag{6}$$

is a free evolution under the secular dipolar Hamiltonian, $H_{dd} = \alpha(S_1^z S_2^z - 1/4(S_1^+ S_2^- + S_1^- S_2^+))$, and $B_x(t)$ is an evolution operator of the system irradiated by the rf. magnetic field

$$B_x(t) = e^{-i\{\omega_1(S_1^x+S_2^x)+H_{dd}\}t}. \tag{7}$$

In (7) one cannot neglect by the dipolar terms. The initial state prepared by a $(\pi/2)_x$ pulse from the thermal state is

$$\rho_{0+} = (1/4)(1+\sigma_1^y) \otimes (1+\sigma_2^y). \tag{8}$$

We assume that the spins are parallel. In another (antiparallel) configuration the contribution to the echo will be zero. In the tilted frame

$$\tilde{B_x}(t) = e^{i\pi/2(S_1^y+S_2^y)}B_x(t_b)e^{-i\pi/2(S_1^y+S_2^y)} = e^{-i\{\omega_1(S_1^z+S_2^z)-H_{dd}/2+3\alpha/8(S_1^+S_2^+ + S_1^-S_2^-)\}t}, \tag{9}$$

where the last term does not commute with S^z. Making ω_1 large, one can reduce effect of this term. The total evolution operator is

$$\hat{U} = e^{-iH_{dd}t_2}e^{-iH't_b}e^{-iH_{dd}t_1}, \tag{10}$$

$$H' = \omega_1(S_1^z + S_2^z) - H_{dd}/2 + 3\alpha/8(S_1^+S_2^+ + S_1^-S_2^-). \tag{11}$$

Now, we change the basis with the unitary transformation

$$\hat{S} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \tag{12}$$

which corresponds to

$$\begin{array}{l} |\uparrow\uparrow\rangle \rightarrow |1\rangle \\ |\downarrow\downarrow\rangle \rightarrow |2\rangle \\ |\uparrow\downarrow\rangle \rightarrow |3\rangle \\ |\downarrow\uparrow\rangle \rightarrow |4\rangle \end{array} \tag{13}$$

In this basis the evolution Hamiltonians are block-diagonal

$$H_{dd} = \begin{pmatrix} (\alpha/4)\hat{\mathbf{1}} & 0 \\ 0 & -(\alpha/4)(\hat{\mathbf{1}} + \sigma_x) \end{pmatrix}, \tag{14}$$

and

$$H' = \begin{pmatrix} -(\alpha/4)\hat{\mathbf{1}} + \omega_1\sigma_z + (3\alpha/8)\sigma_x & 0 \\ 0 & (\alpha/8)(\hat{\mathbf{1}} + \sigma_x) \end{pmatrix}. \tag{15}$$

In the new basis the $(S_1^y + S_2^y)$ operator is

$$\tilde{S}^y = (1/2)\begin{pmatrix} 0 & -i\sigma_z + \sigma_y \\ -i\sigma_z + \sigma_y & 0 \end{pmatrix}. \tag{16}$$

The initial density matrix we also write in the block form as

$$\rho_{0+} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}, \tag{17}$$

where each element corresponds to a matrix 2×2. The echo is

$$\mathrm{Echo} = 1/2(\tilde{S}^y_{12}\rho_{21} + \tilde{S}^y_{21}\rho_{12}). \tag{18}$$

$$\rho_{21}(t) = U_{22}(t)\rho_{21}(0)U_{11}(t)^{-1}, \tag{19}$$

$$U_{22}(t) = e^{i(\alpha/4)(\hat{\mathbf{1}}+\sigma_x)t_2} e^{-i(\alpha/8)(\hat{\mathbf{1}}+\sigma_x)t_b} e^{i(\alpha/4)(\hat{\mathbf{1}}+\sigma_x)t_1} = e^{i(\alpha/4)(\hat{\mathbf{1}}+\sigma_x)(t_1+t_2-t_b/2)}, \tag{20}$$

$$U_{11}(t) = e^{-i(\alpha/4)\hat{\mathbf{1}}t_2} e^{i((\alpha/4)\hat{\mathbf{1}}-\omega_1\sigma_z-(3\alpha/8)\sigma_x)t_b} e^{-i(\alpha/4)\hat{\mathbf{1}}t_1} = e^{-i(\alpha/4)\hat{\mathbf{1}}(t_1+t_2-t_b/2)} e^{-i(\omega_1\sigma_z+(3\alpha/8)\sigma_x)t_b}, \tag{21}$$

$$\rho_{21}(t) = e^{i(\alpha/2)\hat{\mathbf{1}}(t_1+t_2-t_b/2)} e^{i(\alpha/4)\sigma_x(t_1+t_2-t_b/2)} \rho_{21}(0) e^{i(\omega_1\sigma_z+(3\alpha/8)\sigma_x)t_b}. \tag{22}$$

In Rhim's paper the last exponential term was assumed to be $\sim \hat{1}$. Then the only timescale that appears is $t_1 + t_2 - t_b$. Single spin coherence is recovered at $t_2 = t_b - t_1$ The exact evaluation with the initial density matrix $\rho_{12}(0) = -i\sigma_z + \sigma_y$ gives

$$\mathrm{Echo} = \cos(3\alpha/4(t_1 + t_2 - t_b))\cos\Omega t_b + 3\alpha/(8\Omega)\sin(3\alpha/4(t_1 + t_2 - t_b))\sin\Omega t_b, \tag{23}$$

where $\Omega = \sqrt{\omega_1^2 + (3\alpha/8)^2}$ and $\Omega t_b \approx 2\pi n$.

References

[1] Charles P. Slichter, Principles of Magnetic Resonance, Springer Series in Solid State Science, 3-rd edition, Springer Verlag, New York, 2006 (Chapter 8).

[2] Lorenza Viola and Seth Lloyd, Phys. Rev. A **58**, 2733 (1998).

[3] W.-K. Rhim, A. Pines, and J. S. Waugh, Phys. Rev. Lett. **25**, 218 (1970).