# Exclusivity and Control: Online Appendix (Not For Publication)

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## 1 Offer Game

The Stage I bargaining game can be altered such that it is now $C$ who makes a set of offers to each platform:

I. 1. $C$ offers a menu of contracts $T_A = \{T_A^e, T_A^m\}$ and $T_B = \{T_B^e, T_B^m\}$ to each respective platform, where $T_i^e$ is the monetary transfer from platform $i$ to the content provider in exchange for exclusive affiliation, and $T_i^m$ is the transfer when the content provider multihomes.

I. 2. Each platform $i$ simultaneously decides to accept neither, one, or both of the exclusive and non-exclusive components of offered transfers.

I. 3. $C$ chooses which platform(s) to join, where $C$ can join platform $i$ exclusively only if $i$ accepted $T_i^e$, and $C$ can join both platforms only if both platforms each accepted $T_i^m$. Payoffs are as in the bidding game.

Note that by since $C$ makes its offers public and cannot "secretly" affiliate with another platform after promising the other exclusivity, the *opportunism* problem of Hart and Tirole (1990) can be avoided. The results from the offer game are similar to those obtained earlier, as the following proposition shows.

**Proposition 1.1.** *If* $2\Pi_P(0) + \Pi_C^m \leq \Pi_P(1) + \Pi_P(-1) + \Pi_C^e(1)$, *an exclusive equilibrium exists whereby* $C$ *extracts* $T_i^e = \Pi_P(1) - \Pi_P(-1)$ *from the exclusive platform.*

*If* $2\Pi_P(0) + \Pi_C^m \geq \Pi_P(1) + \Pi_P(-1) + \Pi_C^e$, *a multihoming equilibrium exists whereby* $C$ *extracts* $T_i^m = \Pi_P(0) - \Pi_P(-1)$ *from each platform.*

*Proof.* $C$ can induce exclusivity and get the highest payment by setting $T_A = \{\Pi_P(1) - \Pi_P(-1), -\infty\}, T_B = \{\Pi_P(1) - \Pi_P^0 - \varepsilon, -\infty\}$. This will induce both platforms $A, B$ to accept the contracts, and $C$ will be exclusive with $A$.

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$C$ can induce multihoming subject to

$$\begin{aligned} \Pi_C^m + T_i^m + T_j^m &\geq \Pi_C^e(1) + T_i^e \geq 0 \ \forall i \\ \Pi_P(0) - T_i^m &\geq \Pi_P(-1) \ \forall i \end{aligned}$$

The first equation is the (IC) constraint on $C$ (otherwise $C$ would prefer to be exclusive); the second is the (IR) constraint on platforms. Consequently, the highest $C$ can set transfers are $T_i^m = \Pi_P(0) - \Pi_P(-1)$, and to do so $C$ sets $T_i^e \leq \Pi_C^m - \Pi_C^e(1) + 2[\Pi_P(0) - \Pi_P(-1)]$.

Multihoming will an equilibrium if total profits under multihoming are higher than exclusivity; exclusivity will be equilibrium under reverse. Computing $C$'s profits yields the result. ■

# 2 Contracts with royalties

For completeness, the following proposition shows that even with royalties, the Stage I bargaining outcome will still be the industry structure that maximizes industry profits subject to price competition in Stage II, which in turn restricts outcomes from achieving the maximum joint cooperative payoffs.

**Proposition 2.1.** *Allow contracts to take the form $(T_j^e, r_j^e, T_j^m, r_j^m)$, which now include per-unit royalties in addition to fixed fee transfers. An exclusive equilibrium in which the content provider affiliates either with platform A or with platform B always exists; a multihoming equilibrium exists if and only if:*

$$2\Pi_P^m(r_A^{m*}, r_B^{m*}) + \Pi_C^m(r_A^{m*}, r_B^{m*}) \geq \Pi_P^{e+}(r_A^{e*}) + \Pi_P^{e-}(r_A^{e*}) + \Pi_C^e(r_A^{e*}) \quad (2.1)$$

*where $r_j^{e*}$ and $r_j^{m*}$ indicate platform j's equilibrium royalty rate under exclusivity and non-exclusivity, respectively, and are given by:*

$$\begin{aligned} r_j^{e*} &= \arg\max_{r_j^{e'}} \{\Pi_j^{e+}(r_j^{e'}) + \Pi_C^e(r_j^{e'})\} \\ r_j^{m*} &= \arg\max_{r_j^{m'}} \{\Pi_j^m(r_j^{m'}, r_{-j}^{m*}) + \Pi_C^m(r_j^{m'}, r_{-j}^{m*})\} \end{aligned}$$

*Proof.* As in proposition 3.1, clearly an equilibrium with exclusivity always exists.

Assume $\{(r_j^m, T_j^m, r_j^e, T_j^e)\}_{j\in\{A,B\}}$ is an equilibrium whereby $C$ multihomes. Then $C$ receives utility

$$U \equiv \Pi_C^m(r_A^m, r_B^m) + T_A^m + T_B^m = \Pi_C^e(r_j^e) + T_j^e \geq 0$$

where the equality must hold for all $j$ since if this were not the case, either $A$ or $B$ could profitably deviate by reducing $T_i^m$.

The following three conditions must hold as well:

$$r_j^m = r_j^{m*} \equiv \arg\max_{r_j^{m'}} \{\Pi_j^m(r_j^{m'}, r_{-j}^m) - T_j^m\} = \arg\max_{r_j^{m'}} \{\Pi_j^m(r_j^{m'}, r_{-j}^m) + \Pi_C^m(r_j^{m'}, r_{-j}^m)\}$$

which is simply the equilibrium profit maximizing condition for each platform, substituting in the first constraint for $T_j^m$;

$$\Pi_B^m(r_A^m, r_B^m) - T_B^m \geq \Pi_B^{e-}(r_A^e)$$

otherwise $B$ can profitably deviate by refusing to contract with $C$ (i.e., set $T_B^m = -\infty$); and:

$$\Pi_A^m - T_A^m \geq \max_{r_A^{e'}}\{\Pi_A^{e+}(r_A^{e'}) + \Pi_C^e(r_A^{e'}) - U\}$$

or $A$ could profitably deviate by offering a new contact $(r_A^{e'}, T_A^{e'})$ with $T_A^{e'} = U - \Pi_C^e(r_A^{e'})$ which would induce $C$ to be exclusive.

Let $r_A^{e*} = \arg\max_{r_A^{e'}}\{\Pi_A^{e+}(r_A^{e'}) + \Pi_C^e(r_A^{e'})\}$. Adding the previous two inequalities, substituting in the definition of $U$, and rearranging yields the following necessary inequality:

$$\Pi_A^m(r_A^{m*}, r_B^{m*}) + \Pi_B^m(r_A^{m*}, r_B^{m*}) + \Pi_C^m(r_A^{m*}, r_B^{m*}) \geq \Pi_A^{e+}(r_A^{e*}) + \Pi_B^{e-}(r_A^{e*}) + \Pi_C^e(r_A^{e*})$$

which is the condition in the text.

To show that this condition is sufficient for a multihoming equilibrium to exist, note that the equilibrium royalty rates are already provided by $\{r_j^{e*}, r_j^{m*}\}_{j\in\{A,B\}}$. Allowing the first, third, and fourth inequalities in the proof to bind allows for the construction of the equilibrium lump sum transfers $\{T_j^e, T_j^m\}_{j\in\{A,B\}}$. ■

Here, Stage II profits are now a function of the royalty payments contracted upon in Stage I since they influence the prices that will be chosen by platforms and $C$. Enlarging the contracting space does not affect the ability of agents to achieve the joint maximizing profits, but serves to add an additional layer of complexity for analysis.

# 3 Market Expansion

We modify the model of Section 4 to note the possibility that each platform faces a downward sloping demand of "loyal" consumers on each side of the unit interval (i.e., "hinterlands") – these are consumers who will never consider buying the alternative platform. Assume the stand alone utility from getting access to any of the two platforms is $V$ and that loyal consumers for each platform have transportation costs $\frac{2}{u}$, where $u \geq 0$. When the content is exclusive with platform A, if both platforms capture a positive share of the Hotelling segment, then the demand for each platform is given by:

$$D^A = \frac{1}{2}\left(1 + v - P^A + P^B\right) + \frac{\left(V + v - P^A\right)u}{2}$$

$$D^B = \frac{1}{2}\left(1 - v - P^B + P^A\right) + \frac{\left(V - P^B\right)u}{2}$$

Thus, $u > 0$ captures the possibility that the market may possibly expand beyond the covered unit interval. Note that as $u$ gets closer to 0, each platform cares less about attracting its loyal customers and instead will compete more fiercely for the middle segment of customers; this would be equivalent to having no market expansion effect. As $u$ increases, each platform cares less about competing for the middle customer group and therefore the platforms' markets are more independent of each other. Costs for providing both the content and each platform are assumed to be 0.

**Proposition 3.1.** *The market equilibrium depends on the magnitude of market expansion $u$ and the quality of the content $v$ in the following way:*

- *With no market expansion ($u = 0$), exclusivity prevails for all values of $v$;*
- *For positive but sufficiently small market expansion effects ($u < \overline{u}$), there exists a threshold quality level $\overline{v}(u)$ such that multihoming prevails for $v \leq \overline{v}(u)$ and exclusivity prevails for $v > \overline{v}(u)$;*
- *When market expansion effects are large ($u > \overline{u}$), multihoming prevails for all possible quality levels.*

*Proof.* When the content is exclusive with platform A, assuming interior solutions, we obtain the following equilibrium in platform access prices:

$$P^{A*} = \frac{3+2u}{4(1+u)^2-1}(1+Vu) + \frac{2(1+u)^2-1}{4(1+u)^2-1}v$$

$$P^{B*} = \frac{3+2u}{4(1+u)^2-1}(1+Vu) - \frac{1+u}{4(1+u)^2-1}v$$

For this equilibrium to be interior, we need to impose:

$$P^{B*} \geq 0 \iff v \leq \frac{(1+Vu)(3+2u)}{1+u}$$

and:

$$t - v - P^{B*} + P^{A*} \geq 0 \iff v \leq \frac{(1+u)(1+2u)}{4(1+u)^2-1}$$

Platform profits are then:

$$\Pi^A = \Pi_P^{e+} = \frac{1+u}{2\left(4(1+u)^2-1\right)^2}\left[(3+2u)(1+Vu) + \left(2(1+u)^2-1\right)v\right]^2$$

$$\Pi^B = \Pi_P^{e-} = \frac{1+u}{2\left(4(1+u)^2-1\right)^2}\left[(3+2u)(1+Vu) - (1+u)v\right]^2$$

When the content provider multihomes, platform demands are:

$$D^A = \frac{1}{2}\left(1 + u(V+v) - P^A(1+u) + P^B\right)$$

$$D^B = \frac{1}{2}\left(t + u(V+v) - P^B(1+u) + P^A\right)$$

yielding equilibrium prices $P^A = P^B = \frac{1+u(V+v)}{1+2u}$ and profits $\Pi^A = \Pi^B = \Pi^{ne} = \frac{(1+u(V+v))^2(1+u)}{2t(1+2u)^2}$.

Given our solution concept, exclusivity will arise as the equilibrium outcome if and only if $2\Pi^{ne} < \Pi^{e+} + \Pi^{e-}$ which, using the expressions above, is equivalent to:

$$2u\left(3+2u\right)^2\left(1+Vu\right)+F\left(u\right)v \leq 0$$

where:

$$F\left(u\right) = 4u^4 + 8u^3 - 3u^2 - 10u - 2$$

Note therefore that for all $u$ sufficiently small, $F\left(u\right) < 0$, hence the equilibrium outcome is multihoming for $v \in [0, \overline{v}\left(u\right)]$ and exclusivity for $v > \overline{v}\left(u\right)$. When $u$ becomes large enough, $F\left(u\right) > 0$, hence multihoming is the only outcome. ∎

# 4 Substitutable Content Providers

We extend our results to a setting with substitutable content providers. Throughout this extension, we assume there are two content providers, which may compete against each other when present on the same platform. Slightly modifying the notation used in the text, let $\Pi_P(N_i, N_j)$, $\Pi_C^e(N_i, N_j)$ and $\Pi_C^m(N_i, N_j)$ respectively represent the Stage II profits of platform $i$, of a content provider exclusive with platform $i$ and of a content provider who multihomes, when platform $i$ has $N_i$ content providers and the other platform $j \neq i$ has $N_j$ content providers. We assume as is natural that:

$$\begin{aligned}
\Pi_P(2,0) &\geq \Pi_P(2,1) \geq \max\left\{\Pi_P(1,1); \Pi_P(2,2)\right\} \\
\min\left\{\Pi_P(1,1); \Pi_P(2,2)\right\} &\geq \Pi_P(1,2) \geq \Pi_P(0,2) \\
\Pi_C^e(2,0) &\geq \Pi_C^e(2,1)
\end{aligned}$$

Due to the existence of substitution effects across content providers on-board the same platform, Proposition 3.1 from the main text no longer applies. Instead, the following proposition identifies conditions under which the four possible allocations of content providers across platforms – $(2,0), (2,2), (1,1), (2,1)$ – are equilibrium outcomes of the same Stage I bargaining procedure used in the main text. All the proofs are at the end of this section.

**Proposition 4.1.** *There are four distinct allocations of content providers across platforms in which both content providers are active when platforms and content providers are symmetric:*

- Allocation (2,0): *An equilibrium in which both content providers are exclusive to the same platform exists if and only if* $\Pi_C^e(2,0) \geq \Pi_C^e(1,1)$ *or:*

$$2\Pi_P(1,1) + 2\Pi_C^e(1,1) \leq \Pi_P(2,0) + \Pi_P(0,2) + 2\Pi_C^e(2,0) \tag{4.1}$$

- *Allocation (1,1): An equilibrium in which each content provider is exclusive to a different platform exists if and only if* $\Pi_C^e(2,0) \leq \Pi_C^e(1,1)$ *and:*

$$2\Pi_P(1,1) + 2\Pi_C^e(1,1) \geq \Pi_P(2,0) + \Pi_P(0,2) + 2\Pi_C^e(2,0) \tag{4.2}$$

- *Allocation (2,1): A necessary condition for equilibrium in which one content provider multihomes and one is exclusive is:*

$$\Pi_C^e(2,0) + \Pi_C^m(2,2) < \Pi_C^m(2,1) + \Pi_C^e(2,1) \tag{4.3}$$

- *Allocation (2,2): If (4.3) does not hold then an equilibrium in which both content providers multihome exists if and only if:*

$$2\Pi_P(2,2) + 2\Pi_C^m(2,2) \geq \Pi_P(2,0) + \Pi_P(0,2) + 2\Pi_C^e(2,0) \tag{4.4}$$

  *and:*

$$\begin{aligned} \Pi_C^e(2,0) &\geq \Pi_C^e(1,1) \\ &\quad \textit{or} \\ \Pi_P(2,2) + \Pi_C^e(2,0) + \Pi_C^m(2,2) &\geq 2\Pi_C^e(1,1) + \Pi_P(1,1) \end{aligned} \tag{4.5}$$

Let us now specify the model further by assuming the two platforms are located at the extremities of a $[0,1]$ Hotelling segment and that the two content providers are non-strategic.[1] We then introduce the following notation:

- $\pi(2,\delta)$: a content provider's profits *per platform user* when there are two content providers affiliated with that platform
- $s(2,\delta)$: each individual user's *net* surplus on a platform on which there are 2 content providers
- $w(2,\delta) \equiv \pi(2,\delta) + s(2,\delta)$: user *gross* surplus on a platform on which there are 2 content providers
- $\delta$: degree of substitutability (or intensity of competition) between the two content providers; $\pi(2,\delta)$ is *decreasing* and $s(2,\delta)$ is *increasing* in $\delta$
- $\pi(1)$: a content provider's profits *per platform user* when it is the only one content provider affiliated with that platform; assume
- $s(1)$: each individual user's *net* surplus on a platform on which there is 1 content provider

We also make the following assumptions:
**Assumption (A1)** $\pi(1) > \pi(2,\delta)$ *and* $s(2,\delta) > s(1)$
**Assumption (A2)** $1 > \frac{s(2,\delta)}{3} > \frac{s(1)}{3}$
**Assumption (A3)** $\frac{\pi(2,\delta)}{\pi(1)} \geq \frac{1-\frac{s(2,\delta)}{3}+\frac{s(1)}{3}}{1-\frac{s(2,\delta)}{3}+\frac{2s(1)}{3}}$

[1] As observed in the main text of the paper, this can be obtained by assuming content providers set their prices after users make their platform adoption decisions.

Assumptions (A1) and (A2) are natural. (A1) means that a content provider makes higher profits per platform user when it is alone relative to when both content providers are present; conversely, users derive higher net surplus on a platform on which there are more content providers. (A2) ensures that neither platform covers the entire user market even when both content providers are exclusive with it.

Assumption (A3) is made in order to simplify the analysis: it ensures that the $(2,1)$ allocation is not an equilibrium under content affiliation by requiring that competition between content providers is not too fierce.

With this modelling specification we have the following corollaries to Proposition 1 (proofs are at the end of this section):

**Corollary 4.2.** *If content is sold outright then:*

- *the* $(2,0)$ *exclusive allocation is always an equilibrium*
- *the* $(1,1)$ *exclusive allocation is never an equilibrium*
- *the* $(2,1)$ *allocation is never an equilibrium*
- *the* $(2,2)$ *multihoming allocation is never an equilibrium*

**Corollary 4.3.** *Under assumptions (A1)-(A3), if content providers affiliate with platforms then:*

- *the* $(2,0)$ *exclusive allocation is an equilibrium if and only if:*

$$1 \geq \frac{9\left[\pi\left(1\right)-\pi\left(2,\delta\right)\right]}{s\left(2,\delta\right)\left[3\pi\left(2,\delta\right)+s\left(2,\delta\right)\right]}$$

- *the* $(1,1)$ *exclusive allocation is an equilibrium if and only if:*

$$1 \leq \frac{9\left[\pi\left(1\right)-\pi\left(2,\delta\right)\right]}{s\left(2,\delta\right)\left[3\pi\left(2,\delta\right)+s\left(2,\delta\right)\right]}$$

- *the* $(2,1)$ *allocation is never an equilibrium*
- *the* $(2,2)$ *multihoming allocation is an equilibrium if and only if:*

$$1 \leq \frac{9\pi\left(2,\delta\right)}{s\left(2,\delta\right)\left[3\pi\left(2,\delta\right)+s\left(2,\delta\right)\right]}$$

The results for the case of outright sale are the same as in the model from the main text (without substitution between content providers): only the most extreme exclusive equilibrium allocation exists, since it is the one that maximizes vertical differentiation between the two platforms (given that content providers do not make any profits in stage 2).

Consider now the case with content affiliation:

- The $(2,0)$ exclusive equilibrium is more likely when $s(2,\delta)$ and $\pi(2,\delta)$ are larger and $\pi(1)$ is smaller. The intuition is very similar to the one provided in the main text, except that now we have to distinguish between $\pi(2,\delta)$ -which makes exclusivity more likely – and $\pi(1)$ – which makes exclusivity less likely (this is easily understood). Note therefore that $\delta$ has two conflicting effects on the likelihood of exclusivity: by increasing $s(2,\delta)$ it makes it more likely since it increases the scope for vertical differentiation between platforms from users' perspective; on the other hand $\delta$ decreases $\pi(2,\delta)$, which makes exclusivity less likely since the content provider portion of industry profits is lower. Thus, even though the condition is slightly different than (3.2) in the main text, the interpretation is quite similar.

- The necessary and sufficient condition for $(1,1)$ to be an exclusive equilibrium is the opposite relative to $(2,2)$, so the interpretation is simply reversed. Note that a larger $\pi(1)$ makes this allocation more likely to be an equilibrium, as expected.

- Finally, the necessary and sufficient condition for $(2,2)$ to be an equilibrium is exactly the same as 3.2 in the main text (replacing as is natural $\pi(p^m)$ by $\pi(2,\delta)$ and $2s(p^m)$ by $s(2,\delta)$). This multihoming allocation is more likely to arise when $s(2,\delta)$ is smaller and $\pi(2,\delta)$ is larger, i.e. when $\delta$ is smaller. This is easily understood: less substitutability (competition) among content providers makes it less costly in terms of content provider profits to put them together on the same plaform and at the same time it also reduces the scope for vertical differentiation between platforms that can be achieved by seeking exclusivity.

In conclusion, the interpretation of the results with substitutable content providers is largely the same as in the main text. It is worth emphasizing that the fundamental conclusion is unaltered: the change in control rights (i.e. going from outright sale to affiliation) makes the likelihood of exclusivity depend on the division of surplus between consumers and content providers.

## 4.1 Proofs for Substitutable Content

*Proof of Proposition 4.1:*
*Allocation (2,0)*: Suppose first that $\Pi_C^e(2,0) \geq \Pi_C^e(1,1)$. We will construct an equilibrium in which both content providers are exclusive with platform A. Set $T_A^m = T_B^m = -\infty$, which is an equilibrium since multihoming can be ruled out unilaterally by either one of the two platforms: this implies that the only deviations we need to consider are those that have each content provider exclusive to either one of the two platforms. Following the proof of proposition 3.1, the following constraints must hold in equilibrium:

- Content provider incentive compatibility – content providers do not want to switch unilaterally or jointly to be exclusive with platform B:

$$\Pi_C^e(2,0) + T_A^e \geq \max\left\{\Pi_C^e(2,0), \Pi_C^e(1,1)\right\} + T_B^e = \Pi_C^e(2,0) + T_B^e$$

- Platform incentive compatibility:

– Platform B does not want to raise transfers to capture both content providers exclusively (note indeed that $\Pi^e_C(2,0) \geq \Pi^e_C(1,1)$ implies that the allocation $(1,1)$ can never be an equilibrium given any set of transfers $(T^e_i, T^m_i)$ offered by the two platforms, thus the only possible deviation by B is to attract both content providers exclusively):

$$\Pi_P(0,2) \geq \Pi_P(2,0) - 2T^e_A$$

– Platform A does not wish to lower transfers and get no content providers:

$$\Pi_P(2,0) - 2T^e_A \geq \Pi_P(0,2)$$

Let then $T^e_A = T^e_B = \frac{\Pi_P(2,0)-\Pi_P(0,2)}{2}$. It is then clear that, together with $T^m_A = T^m_B = -\infty$, these transfers support the desired equilibrium (in which both platforms make profits $\Pi_P(0,2) \geq 0$ and content providers make $\Pi^e_C(2,0) + \frac{\Pi_P(2,0)-\Pi_P(0,2)}{2} > 0$).

Suppose now that $\Pi^e_C(2,0) \leq \Pi^e_C(1,1)$. We start in the same way with $T^m_A = T^m_B = -\infty$. The constraints that must hold in equilibrium become:

- Content provider incentive compatibility:

$$\Pi^e_C(2,0) + T^e_A \geq \Pi^e_C(1,1) + T^e_B \tag{4.6}$$

- Platform incentive compatibility:

  – Platform B does not want to raise transfers to capture exactly one content provider exclusively. The minimum transfer which allows it to do that is $T^e_B = T^e_A + \Pi^e_C(2,0) - \Pi^e_C(1,1)$: note indeed that with such a transfer the only possible equilibrium allocation is $(1,1)$. Thus, we need to have:

  $$\Pi_P(0,2) \geq \Pi_P(1,1) - T^e_A - \Pi^e_C(2,0) + \Pi^e_C(1,1)$$

  or:

  $$T^e_A \geq \Pi_P(1,1) - \Pi_P(0,2) + \Pi^e_C(1,1) - \Pi^e_C(2,0) \tag{4.7}$$

  – Platform B does not want to raise transfers to capture both content providers exclusively. To do that, the minimal required transfer is $T^e_B = T^e_A + \Pi^e_C(1,1) - \Pi^e_C(2,0)$, so that the relevant constraint is:

  $$\Pi_P(0,2) \geq \Pi_P(2,0) - 2T^e_A - 2\left[\Pi^e_C(1,1) - \Pi^e_C(2,0)\right]$$

  or:

  $$T^e_A \geq \frac{\Pi_P(2,0) - \Pi_P(0,2)}{2} - \left[\Pi^e_C(1,1) - \Pi^e_C(2,0)\right] \tag{4.8}$$

  – Platform A does not wish to lower its transfer and cede both content providers to B exclusively:

  $$\Pi_P(2,0) - 2T^e_A \geq \Pi_P(0,2)$$

  or:

  $$T^e_A \leq \frac{\Pi_P(2,0) - \Pi_P(0,2)}{2} \tag{4.9}$$

– Platform A does not wish to lower its transfer in order to cede exactly one content provider to B (exclusively):

$$\Pi_P(2,0) - 2T_A^e \geq \Pi_P(1,1) - T_B^e - \Pi_C^e(1,1) + \Pi_C^e(2,0)$$

Combining the last inequality with (4.6) we obtain the following necessary condition:

$$T_A^e \leq \Pi_P(2,0) - \Pi_P(1,1) \tag{4.10}$$

Suppose that $\Pi_P(2,0) + \Pi_P(0,2) < 2\Pi_P(1,1)$. Then (4.7) and (4.10) are impossible to satisfy at the same time, therefore the $(2,0)$ allocation cannot be sustained as an equilibrium in this case.

If on the other hand $\Pi_P(2,0) + \Pi_P(0,2) \geq 2\Pi_P(1,1)$ then the necessary constraints to satisfy for $(2,0)$ to be an equilibrium are (4.7), (4.8) and (4.9), which can be satisfied if and only if:

$$\Pi_P(2,0) + \Pi_P(0,2) + 2\Pi_C^e(2,0) \geq 2\Pi_P(1,1) + 2\Pi_C^e(1,1)$$

This condition is more stringent than $\Pi_P(2,0) + \Pi_P(0,2) \geq 2\Pi_P(1,1)$ when $\Pi_C^e(2,0) \leq \Pi_C^e(1,1)$, therefore it is the only necessary condition for $(2,0)$ to be an equilibrium when $\Pi_C^e(2,0) \leq \Pi_C^e(1,1)$. Conversely, if these two conditions are satisfied, then it is easily seen that $T_A^e = \frac{\Pi_P(2,0)-\Pi_P(0,2)}{2}$ and $T_B^e = T_A^e + \Pi_C^e(2,0) - \Pi_C^e(1,1)$ sustain the $(2,0)$ allocation as an equilibrium ($T_A^m = T_B^m = -\infty$).

*Allocation (1,1)*: For both content providers to be exclusive to different platforms, we can again set $T_A^m = T_B^m = -\infty$. The IC constraints for the the content providers are now:

$$\Pi_C^e(1,1) + T_i^e \geq \Pi_C^e(2,0) + T_{-i}^e \qquad \text{for } i \in \{A,B\}$$

These two inequalities can hold only if $\Pi_C^e(1,1) \geq \Pi_C^e(2,0)$ and $|T_A^e - T_B^e| \leq \Pi_C^e(1,1) - \Pi_i^e(2,0)$. Assume therefore that $\Pi_C^e(1,1) \geq \Pi_C^e(2,0)$.

The necessary IC constraints for the platforms are then:

$$\begin{aligned}\Pi_P(1,1) - T_i^e &\geq \Pi_P(0,2)\\ \Pi_P(1,1) - T_i^e &\geq \Pi_P(2,0) - 2\left[T_{-i}^e + \Pi_C^e(1,1) - \Pi_C^e(2,0)\right] \qquad \text{for } i \in \{A,B\}.\end{aligned}$$

These inequalities can be re-expressed as:

$$\begin{aligned}T_i^e &\leq \Pi_P(1,1) - \Pi_P(0,2) \qquad \text{for } i \in \{A,B\}\\ T_i^e - 2T_{-i}^e &\leq \Pi_P(1,1) - \Pi_P(2,0) + 2\left[\Pi_C^e(1,1) - \Pi_C^e(2,0)\right] \qquad \text{for } i \in \{A,B\}.\end{aligned}$$

They imply:

$$|T_A^e - T_B^e| \leq 2\Pi_P(1,1) - \Pi_P(0,2) - \Pi_P(2,0) + 2\left[\Pi_C^e(1,1) - \Pi_C^e(2,0)\right]$$

Consequently, the $(1,1)$ allocation is an equilibrium only if $\Pi_C^e(1,1) \geq \Pi_C^e(2,0)$ and:

$$\Pi_P(2,0) + \Pi_P(0,2) + 2\Pi_C^e(2,0) \leq 2\Pi_P(1,1) + 2\Pi_C^e(1,1)$$

Conversely, if these two conditions hold, it is easily seen that $(1,1)$ can be sustained as an equilibrium with $T_A^e = T_B^e = \Pi_P(1,1) - \Pi_P(0,2)$.

*Allocation (2,1)*: Assume 1 content provider joins both platforms, and 1 joins platform $A$ exclusively. For this to be an equilibrium, the following conditions must hold:

- Content multihoming IC:

$$\Pi_C^m(2,1) + T_A^m + T_B^m \geq \begin{cases} \Pi_C^e(2,0) + T_A^e \\ \Pi_C^e(1,1) + T_B^e \\ 0 \end{cases} \tag{4.11}$$

- Content exclusive IC:

$$\Pi_C^e(2,1) + T_A^e \geq \begin{cases} \Pi_C^m(2,2) + T_A^m + T_B^m \\ \Pi_C^e(2,1) + T_B^e \\ 0 \end{cases} \tag{4.12}$$

Combining the first constraint in (4.11) with the first constraint in (4.12), we obtain the following necessary condition:

$$\Pi_C^e(2,0) - \Pi_C^m(2,1) \leq T_A^m + T_B^m - T_A^e \leq \Pi_C^e(2,1) - \Pi_C^m(2,2)$$

This can only be possible if:

$$\Pi_C^m(2,1) + \Pi_C^e(2,1) \geq \Pi_C^m(2,2) + \Pi_C^e(2,0)$$

*Allocation (2,2)*: The following conditions are necessary and sufficient for an equilibrium where both content providers multihome:

- Content incentive compatibility:

$$\Pi_C^m(2,2) + T_A^m + T_B^m \geq \max\{\Pi_C^e(1,1), \Pi_C^e(2,0)\} + T_i^e \qquad \text{for } i \in \{A,B\} \tag{4.13}$$

- Platform incentive compatibility – neither platform wants to deviate and attract content providers exclusively (which it can do by setting $T_i^m = -\infty$ and offering a sufficient exclusive payment) or none at all:

$$\Pi_P(2,2) - 2T_i^m \geq \begin{cases} \Pi_P(2,0) - 2(\max\{\Pi_C^e(1,1) - \Pi_C^e(2,0), 0\} + T_{-i}^e) \\ \Pi_P(1,1) - (\Pi_C^e(2,0) + T_{-i}^e - \Pi_C^e(1,1)) \text{ only relevant if } \Pi_C^e(2,0) \leq \Pi_C^e(1,1) \\ \Pi_P(0,2) \end{cases} \tag{4.14}$$

  Furthermore, neither platform can lower $T_i^m$ and still induce all content providers to multihome, which is equivalent to requiring that (4.13) is binding.

- Note that, since condition (4.3) does NOT hold by assumption, neither platform can deviate to the $(2,1)$ allocation, which is not an equilibrium.

If $\Pi_C^e(2,0) \geq \Pi_C^e(1,1)$, we can combine (4.13) and the first equation of (4.14) to obtain:

$$\frac{1}{2}(\Pi_P(2,2) - \Pi_P(2,0) \geq T_i^m - T_{-i}^e \geq \Pi_C^e(2,0) - \Pi_C^m(2,2) - T_j^m$$

which, combined with the last inequality of (4.14) ($2T_j^m \leq \Pi_P(2,2) - \Pi_P(0,2)$) yields (4.4), the condition in the proposition.

If $\Pi_C^e(2,0) < \Pi_C^e(1,1)$, the same combination of inequalities also yields (4.4), however there is now the additional constraint given by the second inequality in (4.14 ) which can be re-expressed as: $T_i^m \leq (\Pi_P(2,2) - \Pi_P(1,1) + \Pi_C^e(2,0) - \Pi_C^e(1,1) + T_{-i}^e)/2$. Combining this with (4.13) ($T_A^m + T_B^m \geq \Pi_C^e(1,1) - \Pi_C^m(2,2) + T_i^e$ ), we obtain:

$$\Pi_P(2,2) + \Pi_C^e(2,0) + \Pi_C^m(2,2) \geq 2\Pi_C^e(1,1) + \Pi_P(1,1)$$

which is condition (4.5) given in the proposition. ■

*Proof of Corollary 4.2*

When content is sold outright, we have: $\Pi_C^e(i,j) = \Pi_C^m(i,j) = 0$ for all $i, j \in \{0,1,2\}$; $\Pi_p(1,1) = \Pi_p(2,2) = \frac{1}{2}$; $\Pi_p(2,0) = \frac{1}{2}\left(1 + \frac{w(2,\delta)}{3}\right)^2$ and $\Pi_p(2,0) = \frac{1}{2}\left(1 - \frac{w(2,\delta)}{3}\right)^2$. Then:

- condition (4.1) is equivalent to $1 \leq 1 + \frac{w(2,\delta)^2}{9}$ which always holds;
- condition (4.2) is equivalent to $1 \geq 1 + \frac{w(2,\delta)^2}{9}$ which never holds;
- condition (4.3) is equivalent to $0 < 0$ which never holds;
- condition (4.4) is equivalent to $1 \geq 1 + \frac{w(2,\delta)^2}{9}$ which never holds.

■

*Proof of Corollary 4.3* If content providers affiliate, we have:

$$\Pi_p(1,1) = \Pi_p(2,2) = \frac{1}{2}$$

$$\Pi_p(2,0) = \frac{1}{2}\left(1 + \frac{s(2,\delta)}{3}\right)^2; \ \Pi_p(2,0) = \frac{1}{2}\left(1 - \frac{s(2,\delta)}{3}\right)^2$$

$$\Pi_C^e(2,0) = \pi(2,\delta)\frac{1}{2}\left(1 + \frac{s(2,\delta)}{3}\right)$$

$$\Pi_C^e(1,1) = \pi(1)\frac{1}{2}; \ \Pi_C^m(2,2) = \pi(2,\delta)$$

$$\Pi_C^e(2,1) = \pi(2,\delta)\frac{1}{2}\left(1 + \frac{s(2,\delta) - s(1)}{3}\right)$$

$$\Pi_C^m(2,1) = \frac{\pi(2,\delta)}{2}\left(1 + \frac{s(2,\delta) - s(1)}{3}\right) + \frac{\pi(1)}{2}\left(1 - \frac{s(2,\delta) - s(1)}{3}\right)$$

First, note that condition the necessary condition (4.3) for the allocation $(2,1)$ to be an equilibrium is equivalent to $\frac{\pi(2,\delta)}{\pi(1)} < \frac{1 - \frac{s(2,\delta)}{3} + \frac{s(1)}{3}}{1 - \frac{s(2,\delta)}{3} + \frac{2s(1)}{3}}$, which means that assumption (A3) rules out allocation $(2,1)$ as an equilibrium.

Second, the $(2,0)$ exclusive allocation is an equilibrium if and only if:

$$\pi(2,\delta)\left(1+\frac{s(2,\delta)}{3}\right)\geq\pi(1)$$

or:

$$\pi(1)\leq\frac{s(2,\delta)^2}{9}+\pi(2,\delta)\left(1+\frac{s(2,\delta)}{3}\right)$$

i.e. if and only if the last inequality holds (since it is implied by the first one). And the last inequality can also be written:

$$1\geq\frac{9\left[\pi(1)-\pi(2,\delta)\right]}{s(2,\delta)\left[3\pi(2,\delta)+s(2,\delta)\right]} \tag{4.15}$$

Third, the $(1,1)$ exclusive allocation is an equilibrium if and only if the opposite inequality (relative to 4.15) holds.

Fourth, the $(2,2)$ multihoming allocation is an equilibrium if and only if:

$$1\leq\frac{9\pi(2,\delta)}{s(2,\delta)\left[3\pi(2,\delta)+s(2,\delta)\right]}\text{ (which corresponds to 4.4)}$$

and

$$\begin{aligned}\pi(2,\delta)\left(1+\frac{s(2,\delta)}{3}\right) &\geq \pi(1)\ \text{ (this corresponds to } \Pi_C^e(2,0)\geq\Pi_C^e(1,1)\text{ )}\\ &\text{or}\\ \pi(2,\delta)\left(\frac{3}{2}+\frac{s(2,\delta)}{6}\right) &\geq \pi(1)\ \text{ (this corresponds to 4.5)}\end{aligned}$$

But $1>\frac{s(2,\delta)}{3}$ and $s(2,\delta)>s(1)$(assumptions A1 and A2) imply that:

$$\frac{1-\frac{s(2,\delta)}{3}+\frac{s(1)}{3}}{1-\frac{s(2,\delta)}{3}+\frac{2s(1)}{3}}\geq\frac{1}{\frac{3}{2}+\frac{s(2,\delta)}{6}}$$

which means that assumption (A3) guarantees that 4.5 holds. Thus, the necessary and sufficient condition for $(2,2)$to be an equilibrium allocation under (A1)-(A3) is $1\leq\frac{9\pi(2,\delta)}{s(2,\delta)[3\pi(2,\delta)+s(2,\delta)]}$.

■