Competing Platforms

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Abstract

Why do competing platforms or networks exist? This paper focuses on instances where the value of a platform depends on the adoption decisions of a small number of firms, and analyzes the strategic competition among platforms to get this oligopolistic side on-board. I study a bilateral contracting game among platforms and firms that allows for general externalities across both contracting and non-contracting partners, and examine when a market will sustain a single or multiple platforms. When firms can join only one platform, I provide conditions under which market-tipping and/or market-splitting equilibria may exist. In particular, even without coordination failure, congestion effects, or firm multi-homing, multiple platforms can sustain in equilibrium despite being inefficient from the perspective of the contracting parties. Expanding the contracting space to include contingent contracts may exacerbate this inefficiency.

KEYWORDS: Platform competition, two-sided markets, bilateral contracting

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1 Introduction

Many important economic markets conduct transactions between agents facilitated by a "platform" or "network." Examples are numerous, and include technology standards and business exchanges which enable interactions between firms, as well as markets which allow consumers to access the goods and services of other firms.[1] In certain cases, market fragmentation across multiple platforms may not be surprising nor warrant concern, particularly when platforms are sufficiently differentiated.[2] However, when there are strong network effects and externalities that favor agglomeration on a single platform (Katz and Shapiro, 1985), the co-existence of multiple platforms may result in socially inefficient replication of costs and investment, delayed adoption of new technology due to uncertainty, and welfare losses due to restricted choice stemming from product incompatibility.[3]

Though the reason for multiplicity likely varies from industry to industry, potential explanations include coordination failure (c.f. Farrell and Klemperer (2007) for a survey); congestion effects (Ellison and Fudenberg (2003); Ellison, Fudenberg, and Möbius (2004)); and the ability for agents to join multiple platforms or "multi-home". To some extent, these analyses are incomplete as they are frequently predicated on one or both of the following assumptions: (i) agents are atomistic price takers who cannot individually influence market outcomes; and (ii) platforms are non-strategic, acting merely as options for others to choose among. Yet, in many prominent platform markets, neither of these assumptions hold. First, some agents in the market are oligopolistic, strategic, and can individually influence profits and shares of all others; second, since a platform's success will rely on its ability to initially attract these agents to join, they act strategically and actively by offering contracts and other incentive schemes. In these settings, it is no longer clear that the same forces previously identified in the literature lead to the existence of a single platform ("market-tipping") or multiple platforms ("market-splitting").

In an attempt to shed light on this issue, this paper explicitly focuses on the underlying contracting game between platforms and an oligopolistic set of agents, and emphasizes how features of this particular form of competition alone can dictate whether or not a market will tip or sustain multiple platforms. I focus on a setting where agents are symmetric and can only join a single platform (which may be reasonable for markets where the costs of multi-homing are large, such as the adoption of a particular standard or technology, or decision to franchise for a particular brand), and show that even without coordination failure, congestion, or multi-homing, outcomes that are worse for the contracting parties (e.g., multiplicity instead of agglomeration, or vice versa) may persist in equilibria simply as an outcome of platform competition and contracting with externalities.[4]

[1] E.g., hardware-software markets (computer operating systems, videogame consoles), retail marketplaces (shopping centers, auction sites, franchises), content delivery systems (television, online music), and healthcare (managed care organizations).

[2] Heterogeneous consumers may prefer different vertical (price, quality) characteristics or horizontal (location) attributes. Also, platforms may choose to subsidize different sides of a multi-sided market as in Ambrus and Argenziano (2008).

[3] C.f. Farrell and Saloner (1985); empirical work documenting welfare losses due to incompatibility across platforms include Ohashi (2003) on VCRs; Rysman (2004) on yellow pages; Ho (2006) on insurer-hospital networks; Ishii (2005) and Knittel and Stango (2011) on ATM networks; and Lee (2012) in hardware-software markets.

[4] Though a complete analysis allowing for multihoming and asymmetric firms is beyond the scope of our paper,

This paper examines a two-stage bilateral contracting game: platform providers first make offers to a set of oligopolistic agents—which we will refer to as "firms"—that specify the payment made to (or demanded of) each firm conditional on that firm joining the platform; firms then simultaneously choose which platform to affiliate with. I focus on the strategic game played between only these two sets of players—the platforms and firms—even if there may exist other sets of players in the market. For tractability, I assume the eventual market structure and expected payoffs to all parties are determined solely by the contracting decisions of the oligopolistic set of agents.[5] These "partition" or "value" functions—payoffs to agents conditional on the realized network structure—are allowed to be quite general, and will be assumed to be primitives of the analysis.

Since payoffs to any platform or firm will depend on the actions of all agents, the setting analyzed is a multilateral contracting environment with externalities; as such, previous analyses of bilateral contracting games are not directly applicable.[6] I focus on an environment with only two platforms and assume all firms are symmetric and must choose a single platform to join; secondly, I impose the solution concept of Coalition-Proof Nash Equilibrium (CPNE) introduced by Bernheim, Peleg, and Whinston (1987), which avoids coordination problems among firms as well as selects a unique equilibrium outcome for any subgame following the first stage. The setup also avoids employing undesirable restrictions on the beliefs of firms following a deviation by a platform, such as imposing "passive-beliefs" (Hart and Tirole, 1990; McAfee and Schwartz, 1994).

I then analyze two separate cases: first, when platforms are able to offer *contingent* contracts, whereby transfers between platforms and firms depend on the number of firms that join each platform; and secondly, when platforms can only contract on whether or not a firm joins. In the first case, the efficient outcome—defined in our setting to be the outcome which maximizes total surplus among the contracting parties—need not be sustainable as an equilibrium outcome as the inability for platforms to offer side payments to each other can lead to "excessive" competition for firms: e.g., there may be multiple market-splitting equilibria, even if the efficient outcome dictates market-tipping (all firms joining a single platform). If, however, transfers cannot be made contingent on the actions of other firms as in the second case, then market-tipping is more likely to be the only equilibrium outcome. I show that this also applies when there is a continuum of firms.

This result may not be surprising, as it can be seen as a direct parallel of Segal (1999) to the setting with multiple "principals" (platforms): an inefficient outcome from the perspective of contracting parties may arise when there are externalities imposed by a contract on those not in-

insights from this paper still apply to these richer environments. E.g., Lee and Fong (2012) study a model of network formation and bargaining in more general networked industries where asymmetric agents can have multiple contracting partners, and find inefficient outcomes can persist.

[5]If there are only firms and platforms in the market, this assumption is not restrictive. In other settings when there are other agents that act after platforms and firms have contracted, I assume there is a unique subgame equilibrium following the contracting stage. I discuss this assumption further in the next section.

[6]Many of these papers have analyzed a non-cooperative setting where one set of players (principals) typically make take-it-or-leave-it contract offers to the other set of players (agents) in order to induce them to take certain actions. The seminal paper by Bernheim and Whinston (1986) analyzed common agency, with many principals attempting to influence the action of a single agent via transfers; Segal (1999) and Segal and Whinston (2003) studied the case with one principal and many agents; and Prat and Rustichini (2003) explored the case with multiple principals and multiple agents, but did not allow for agents to exert externalities on other agents.

volved. However, what is new to this paper is the result that if efficiency dictates tipping, allowing for contingent contracts may actually lead to market-splitting equilibria whereas prohibiting them may lead to tipping—i.e., expanding the contracting space results in a worse outcome. Key to this result is that (i) transfers between players on the same side (here, between the two platforms or between the firms) and (ii) transfers between firms and platforms that do not contract are often prohibited. Such forced market segmentation, often imposed in these bilateral contracting environments for antitrust or other institutional reasons, prevent the realization of efficient outcomes for the contracting partners.[7]

This result echoes findings in recent empirical work: e.g., Lee (2012) has shown that the presence of exclusive contingent-contracts in a hardware-software market encouraged the existence of multiple platforms, and banning these types of contracts may have led to a monopolist platform provider. Furthermore, there is evidence that exclusive deals may have aided platform entry in certain media markets,[8] or encouraged and enabled certain standards battles.[9] However, though the outcomes may have been inefficient from the perspective of the contracting parties (i.e., reduced industry surplus due to cost replication, delayed adoption, and competition), they may not have been socially inefficient, as consumers may have benefits from lower prices and increased product variety arising from platform competition.

Although the *two-sided markets* literature analyzes platform markets (e.g., Armstrong (2006); Caillaud and Jullien (2003); Rochet and Tirole (2003, 2006)), the focus has mainly been on platform pricing, and in settings where only platforms act strategically and other sides of the market are price-takers. Insofar that some of these papers analyze competing platforms, they do so taking the existence of multiple platforms as given. This paper, on the other hand, attempts to endogenize their existence by modeling this competition for a small number of strategic firms on one side who interact with the platform providers. As a result, it is more similar to the bilateral contracting literature mentioned above and papers on endogenous network formation (e.g., Jackson and Wolinsky (1996); Bloch and Jackson (2007); Kranton and Minehart (2000, 2001)).

The paper concludes by applying the model to an example of marketplace competition, where two competing marketplaces compete for N differentiated product retailers to join their respective sites. Consistent with intuition and previous analysis, the model predicts that strong platform differentiation, weak network effects, or decreasing returns from additional contracting partners makes it less likely for complete market-tipping to be an equilibrium.

[7] Bloch and Jackson (2007) discuss how allowing for these kinds of general transfers restores efficiency in a related network formation game.

[8] E.g., DirecTV's success in television distribution against cable is often partially attributed to its ability to engage in exclusive deals with content providers.

[9] E.g., the recent battle between next-gen DVD formats Blu-ray and HD-DVD was partially spurred on by Sony's exclusive relationship with movie studio Columbia Pictures, and Toshiba's exclusive deal with Paramount.

2 Model

2.1 Setup and Timing

Consider a market with two platforms, A and B, and N symmetric firms in a multi-stage game. I assume firms must affiliate with one platform, and cannot join multiple platforms. The timing of the game is as follows:

(1) In the first stage, each platform i simultaneously offers a transfer schedule t_i. If firms can offer contingent transfers, then $t_i : \{1, ..., N\} \to \mathbb{R}$ is a function which indicates the payment made by platform i to each firm who joins i, given the number of firms who join platform A. For example, if x firms join platform A, platform A pays each of the x firms the amount $t_A(x)$, whereas platform B pays of the $N - x$ firms that join it the amount $t_B(x)$. If firms cannot offer contingent transfers, then $t_i \in \mathbb{R}$ is a single payment specifying what platform i will pay any firm in exchange for affiliation.

(2) In the second stage, each firm simultaneously chooses which platform to join. Let η_i denote the number of firms that join platform i—i.e., if x firms join platform A, $\eta_A = x$ and $\eta_B = N - x$.

(3) Payoffs to all agents are realized: each platform i receives $G_i(x) \equiv V_i(x) - \eta_i t_i(x)$, and each firm that joins platform i receives $F_i(x) \equiv U_i(x) + t_i(x)$. V_i and U_i are assumed to be real valued functions.

One key assumption in our analysis is that these payoff functions can be expressed solely as functions of the actions taken in the first two stages of the game: the transfer schedules $\{t_A(\cdot), t_B(\cdot)\}$ and the number of firms that join each platform $\{\eta_A, \eta_B\}$. In certain markets (e.g., "one-sided" networks like technology standards), firms and platforms are the only relevant agents, and this assumption is without loss of generality. However, in other markets (e.g., "multi-sided" markets), there may be others agents (e.g., consumers) whose actions influence payoffs as well. In these more general settings, I assume firms act first (as in Hagiu (2006)), and there is a unique equilibrium or fixed equilibrium selection rule for any subsequent subgame following firms' contracting decisions. This is a strong assumption in that it rules out potential coordination failures among other agents, and is made to primarily focus on contracting issues between oligopolistic firms and platforms.

Nonetheless, there are multi-sided market settings in which this assumption holds. For example, consider settings in which after firms contract with platforms in stage 2, there are two additional substages prior to payoffs being realized in stage (3): (2a) firms engage in price competition for (an atomless set of) consumers, and (2b) consumers then choose which platforms to join. If there are only indirect network effects for consumers when making their adoption decisions (so that consumers care only about prices and the set of firms on each platform, and not the number of consumers on each platform), then there is no coordination problem among consumers in (2b) and there is a unique allocation of consumers across platforms given prices and firm contracting decisions. Consequently, as long as there is then a unique equilibrium of the price setting subgame among firms and platforms in stage (2a), final payoffs to firms and platforms in (3) can be specified as a function of firm contracting decisions alone. This timing and setup has frequently been used to study platform markets in applied work.[10] I also provide an example in Section 4 where V_i and U_i, generated via a pricing subgame among firms and a CES consumer demand system, are functions solely of the number of firms that join each platform.

For now, I take V_i and U_i as primitives for the analysis and allow them to be as general as possible, subject only to the following two assumptions.

Assumption 2.1 (Positive Surplus)**.** $V_i(x) + \eta_i U_i(x) \geq 0 \; \forall \; i \in \{A, B\}, x \in \{1, ..., N\}$.

Since all firms must join a platform, I will assume that the total surplus generated between firms and platforms is always positive, no matter how many or how few firms join a given platform. Notice that this assumption says nothing about the signs of V_i or U_i, and it says nothing about the division of surplus. In general, one may expect V_A (or V_B) to be increasing in x (or $N-x$)—as more firms are on a platform, there is greater demand for that platform, and V_i may be a monotonic function of demand. U_i however need not be strictly increasing nor decreasing. As a firm who is currently on a particular platform i, there are often two competing effects when another firm joins the same platform: an *inter-platform* effect which is positive, since an additional firm on the same platform makes that platform more attractive to consumers, thereby increasing demand and profits for all firms affiliated; and an *intra-platform* effect which is negative, resulting from increased competition within the platform due to the additional firm.

Assumption 2.2 (Zero Surplus without Firms)**.** $V_A(0) = V_B(N) = 0$.

This normalizing assumption is made for expositional purposes only, and indicates that platforms receive 0 profits if no firms join.

Finally, let x^e denote the outcome which maximizes the sum of platform and firm profits:

$$x^e \in \arg \max_{x \in \{0,...,N\}} V_A(x) + xU_A(x) + V_B(x) + (N-x)U_B(x) \; . \tag{2.1}$$

Though not necessary for the analysis, I will assume that x^e is unique for expositional clarity (which will generally be the case if platforms are asymmetric). If firms and platforms are the only relevant agents, it will coincide with the welfare maximizing outcome; otherwise, one needs to completely specify utilities for other players in the market (e.g,. consumers) and how any other subgames are structured in order to specify the socially efficient outcome. The precise conditions under which productive and social efficiency coincide or diverge will be dependent on the underlying primitives

[10] Examples include insurance companies contracting with hospitals (Ho, 2009; Lee and Fong, 2012), cable television operators contracting with channels (Crawford and Yurukoglu, 2012), and videogame consoles contracting with software providers (Lee, 2012); these as well as many other papers model the firms onboard a platform as characteristics of that platform, and uses a multinomial logit demand system to model consumer demand for each platform. Logit demand systems also admit a unique pricing equilibrium in (2a) under duopoly (as assumed in this paper; c.f. Anderson and de Palma (1988)) and other settings (c.f. Caplin and Nalebuff (1991); Vives (1999); Allon, Federgruen, and Pierson (2011)).

and institutional details of the particular market being analyzed. For the rest of this paper, I will refer to x^e as the efficient outcome even though it will be efficient only from the perspective of the contracting parties.

2.2 Equilibrium

Let $T \equiv T_A \times T_B$ denote the set of allowable transfer schedules for the platforms. If firms can only offer a contract to firms specifying payment for affiliation (i.e., non-contingent contracts), then $T_i \equiv \mathbb{R}$; otherwise, with contingent contracts which specify the amount to be paid to each firm given the number of firms that join, $T_i \equiv \mathbb{R}^N$.[11]

An appropriate solution concept is required to analyze the first two stages of this game. Though using Nash Equilibrium might seem reasonable, it is problematic for several reasons. First, in games with network externalities, using Nash Equilibrium often results in a multiplicity of equilibria. Additionally, it allows outcomes which are "unstable" in the following sense: platform choice is rarely a permanent decision by firms as there is usually substantial movement and adjustment after firms make their initial choices; considering only the possibility of unilateral deviations rules out coalitional deviations as well as sequential changes, both of which are real possibilities. For example, one might not expect to see a Pareto-dominated Nash Equilibrium outcome in the second stage. Using a concept such as Strong Nash Equilibria (Aumann, 1959) that rejects any equilibria that is not coalitionally stable might be ideal and does address these concerns; however, it is too strong of a restriction for analysis: for certain sets of transfers, there will not exist any Strong Nash Equilibrium in the second stage.

Instead, I will use the solution concept of Coalition-Proof Nash Equilibrium (CPNE) (Bernheim, Peleg, and Whinston, 1987) to refine the set of possible outcomes in the second stage. It is weaker than the notion of Strong Nash Equilibrium in that CPNE only considers coalitional deviations that are not subject to further deviations, as opposed to all possible coalitional deviations. Importantly, however, it admits a unique equilibrium prediction for the second stage of the game given the following technical assumption.

Assumption 2.3 (No Indifference)**.** *Assume firms are never indifferent between affiliating with platform A or B, and break ties in a manner consistent with their symmetric preferences. Specifically, if $F_A(x') = F_B(x'')$ for some x', x'', without loss of generality I will assume that firms prefer to be on platform A if x' firms join A than on platform B if $N - x''$ firms join B.*

This assumption implicitly defines a true preference ordering over choices between platforms for each firm—that is, firms' preferences are given by the function $\succ$, where $A(x') \succ B(x'')$ iff

[11] Note there are no restrictions on the range of transfers: if platforms are not budget constrained and can credibly make large offers, no upper-limit on transfers is not problematic. In Appendix B, I explore imposing a feasibility constraint on transfers. Also, note that platforms may demand a transfer such that firms receive a negative payoff in equilibrium (since firms do not have the option of not joining a platform). Although for analytical tractability I do not explicitly handle a firm's individual rationality (IR) constraint and assume that firms have no choice but to affiliate with some platform, it will be the case that equilibrium transfers will only depend on the relative differences between firm utilities; thus, as long as U_i is sufficiently positive, every firm will receive non-negative payoffs in equilibrium.

$F_A(x') \geq F_B(x'')$ and $A(x') \prec B(x'')$ otherwise. As the following proposition proves, the assumption is necessary only to ensure existence of a unique CPNE in all subgames, and only applies for a non-measurable subset of possible transfer schedules.[12]

Proposition 2.1. *For every $\{t_A, t_B\} \in T$, there exists a unique pure-strategy CPNE $\hat{x}(t_A, t_B)$ in the second stage, whereby $\hat{x}$ firms join platform A and $N - \hat{x}$ firms join platform B.*

All proofs are located in the appendix.

In the first stage, I will utilize Nash Equilibrium to determine the transfers made by platforms. It is reasonable to assume that once platforms make their offers, any subsequent changes to their schedules are impossible (or prohibitively costly); furthermore, since there are only 2 platforms, explicit coordination would be unlikely due to possible legal or institutional restrictions.[13]

Finally, I choose to focus only on pure-strategy equilibria. A pure strategy for a platform i is simply an element of T_i, and a pure strategy for a firm is $s_n : T \to \{A, B\}$. Thus, an equilibrium of this game can be denoted by $\{t_A^*, t_B^*, x^*\}$, where:

- $x^* = \hat{x}(t_A^*, t_B^*)$, as defined in Proposition 2.1,
- $t_i^* \in \arg\max_{t_i \in T_i} G_i(\hat{x}(t_i, t_{-i}^*)) \; \forall \, i \in \{A, B\}$.

That is $\{t_A^*, t_B^*, x^*\}$ is subgame-perfect equilibrium of the two-stage game where each platform i optimizes over T_i holding the other platforms' transfers fixed, anticipating correctly that firms will play the unique CPNE $x^* = \hat{x}(t_A^*, t_B^*)$.

3 Analysis

Before analyzing the model with transfers, it is useful to examine briefly the case if platforms were passive and could not offer transfers. In this case, firms would choose a platform based solely on their $U_i(x)$ functions. By Proposition 2.1, there will be a unique CPNE in this case. Furthermore, as the following lemma shows, usually that outcome will be complete market tipping if we believe firms to prefer being on platforms with a greater number of other firms.

Lemma 3.1. *If U_A is strictly increasing, U_B strictly decreasing, $U_A(N) > U_B(0)$, and if platforms are passive and cannot make transfers, then the unique equilibrium will have all N firms joining platform A.*

This result follows simply from noting that no other outcome would be a CPNE for the firms. Thus, as long as the inter-platform benefits accrued to each firm outweigh the costs from intra-platform competition, a single marketplace will emerge when platforms are passive.

[12] An alternative approach would be to admit only transfer schedules s.t. $F_A(x') \neq F_B(x'') \; \forall \, x'$ and $x'' < x'$.

[13] Utilizing CPNE in this particular 2 player stage game would correspond to selecting only Pareto undominated Nash Equilibria, which may not be justified in this context.

		Firm 2	
		A	B
Firm 1	A	$11, 0$	$4, 6$
	B	$4, 6$	$0, 7$

Figure 1: A game whereby the efficient outcome of both firms joining platform A is not stable. (Payoffs are for platforms, whereas actions are for firms).

3.1 Contingent Transfers

Market tipping. When is complete market tipping—i.e., all N firms joining one platform—a possible equilibrium outcome when firms can offer contingent transfers? The following proposition provides a necessary and sufficient condition: the per-firm surplus on the dominant platform is greater than the surplus generated by one firm deviating to the other platform.

Proposition 3.2.

$$\frac{1}{N}V_A(N) + U_A(N) \geq U_B(N-1) + V_B(N-1) \tag{3.1}$$

is a necessary and sufficient condition for there to be an equilibrium where all N firms join platform A.

Even if $x^e = N$, the efficient outcome of all firms joining platform A need not even be a stable equilibrium outcome: what matters, from equation (3.1), is that B and just one firm cannot earn more than the surplus created from the Nth firm joining A. In other words, complete market tipping can only occur if a platform and the Nth firm joining that platform gain more than the surplus obtained when a single firm joins the other platform. Since A cannot offer more than $V_A(N)/N$ to each firm that joins its platform or else it would earn negative profits, as long as B can profitably steal away one firm, the complete market-tipping outcome is not an equilibrium. This can be illustrated simply in the following example:

Example 3.1. *Consider a game with 2 symmetric firms, where firms receive no base utility ($U_i(\cdot) = 0$). Payoffs to the platforms $V_i(\cdot)$ are given in figure 1.*[14] *Note that the efficient outcome would be $x^e = 2$, whereby both firms choose to join platform A. But by proposition 3.2, $V_A(x)/2 = 5.5 \ngeq V_B(1) = 6$, and thus this cannot be an equilibrium outcome. In other words, A can offer each firm at most $t_A(2) = 5.5$ in order to induce them to both join; however, B would offer $t_B(1) > 5.5$ since this would be profitable to do so—i.e., it could steal away a firm and get positive profits. Also, note that $x = 0$ whereby both firms join B is not an equilibrium either.*

This competition between platforms to get firms on board and their ability to offer transfers leads to different implications than straight location choice models on the part of firms (i.e., passive platforms); furthermore, it shows how bilateral contracting in the presence of externalities—even when allowing for contingent transfers—can fail to achieve efficient outcomes. Key to this result

[14] The payoffs can be explained by the following story: A is better at exploiting complementarities across multiple firms, whereas B is a platform that performs almost as well with just one firm than with two.

is that transfers between the two platforms or between the firms, and transfers between firms and platforms that do not contract, are prohibited. If $x^e = N$ and platform A could make a payment to platform B contingent on all N firms joining A, then N would obtain as an equilibrium outcome. However, such transfers are often not allowed in these platform contexts for legal or institutional reasons.

Consider markets where consumers join platforms to access the goods and services of firms who have already joined a platform. In such markets, a platform's profits can often be represented as an increasing function of consumer demand for that platform, either deriving from either a price charged directly to consumers for access or utilization, or through advertising. Then circumstances which would make market tipping less likely would include the presence of large firms whom unilaterally could induce sufficient numbers of consumers to join their platform, regardless of the actions of other firms—examples include "hit" software or content joining certain hardware or content distribution platforms, or even "star" hospitals choosing a particular insurance plan. Furthermore, as platforms are more differentiated (from the perspective of consumers or other adopters), the condition for market tipping in (3.1) is also less likely to hold: platform B might still be attractive to enough consumers (with just one firm) to ensure $V_B(N-1)$ is sufficiently high.

Market splitting. The next proposition provides condition under which a market-splitting or interior equilibria can occur.

Proposition 3.3. *There exists an interior equilibrium $x^* \in \{1, \ldots, N-1\}$ if and only if the solution $(t_A^*(x^*), t_B^*(x^*))$ to*[15]

$$\begin{aligned} t_A^*(x^*) &= \frac{1}{x^*}\left[V_A(x^*) - V_A(x^*+1) \right. \\ &\qquad \left. + (x^*+1)(U_B(x^*) - U_A(x^*+1) + t_B^*(x^*))\right] , \end{aligned} \tag{3.2}$$

$$\begin{aligned} t_B^*(x^*) &= \frac{1}{N-x^*}\left[V_B(x^*) - V_B(x^*-1) \right. \\ &\qquad \left. + (N-x^*+1)(U_A(x^*) - U_B(x^*-1) + t_A^*(x^*))\right] . \end{aligned} \tag{3.3}$$

satisfies the following two inequalities:

$$V_A(x^*) - x^* t_A(x^*) \geq 0 , \tag{3.4}$$

$$V_B(x^*) - (N-x^*) t_B(x^*) \geq 0 . \tag{3.5}$$

Equations (3.2) and (3.3) are (IC) constraints on the platforms, which ensure that transfers each platform offers are high enough so that the other platform is not induced into deviating, whereas

[15] The solution to (3.2) and (3.3) is

$$\begin{aligned} t_A^*(x^*) &= \frac{1}{N+1}\left[V_A(x+1) - V_A(x) - (N-x)(x+1)(U_B(x) - U_A(x+1)) \right. \\ &\qquad \left. + V_B(x-1) - V_B(x) - (N-x+1)(U_A(x) - U_B(x-1))\right] \end{aligned}$$

with a similar expression for $t_B^*(x^*)$

(3.4) and (3.5) are (IR) constraints to ensure that platforms receive positive profits. The proposition proves that these are necessary and sufficient conditions to construct an interior equilibrium that results in outcome x^*.

This proposition proves that the existence of interior equilibrium depends only on these underlying primitives, and there may in fact be a "plateau" of potential interior equilibria that may be induced. Note that in determining if an interior x^* is an equilibrium, (3.2) and (3.3) only refer to:

- $V_A(x^*) - V_A(x^*+1)$ and $V_B(x^*) - V_B(x^*-1)$, the marginal contribution of an additional firm to each platform;
- $U_A(x^*) - U_B(x^*-1)$ and $U_B(x^*) - U_A(x^*+1)$, the gain to a firm who switches from platform A to B or from B to A.

Thus, the (IC) constraints consider only the differences and not absolute values of these primitives (i.e., the potential gains to deviation from x^* for each player). However, the two (IR) constraints given by(3.4) and (3.5) depend only on the absolute levels of $V_i(x^*)$. Consequently, as long as the absolute levels of V_i are sufficiently high compared to the relative gains from a single firm deviating, it is feasible that *all* interior market outcomes could be equilibria. For comparison, recall that under the conditions of Lemma 3.1, an interior equilibrium could not exist when platforms cannot offer transfers. This is no longer the case here, and again this competition between platforms to get firms on-board can induce equilibria that may not have been possible otherwise.

The greater the marginal contribution of a firm to a platform's utility or the difference in utility of a firm by switching platforms in relation to a platform's absolute utility, the less chance that a particular market splitting equilibrium $x^* \in \{1, ..., N-1\}$ can occur. This would occur as the impact of a single firm's decision either on platform profits or own profits increase, which would happen if firms were few in number and/or each commanded significant demand share among consumers (again, if platforms served as intermediaries between firms and consumers). On the other hand, if platforms realized large profits initially from a few firms joining but those benefits fell as more firms joined such that $(V_i(x) - V_i(x-1)) \, / \, (V_i(x))$ is low for intermediate market splits, then many possible interior outcomes are supportable in equilibrium.

3.2 Non-Contingent Transfers

If platforms are restricted to making non-contingent offers, they lose a means of controlling the number of firms that join their own platform. Indeed, the market outcome will not depend on a platform's value function V_i as much as before; rather, only the firm's value function U_i will be crucial.

I analyze two different cases, each of which characterizes a wide class of U_i functions.

Assumption 3.1. *(Network Effects):* $\max_x(U_A(x)) = N$ *and* $\max_x(U_B(x)) = 0$.

Assumption 3.1 may be extreme, as the firm optimal outcome may not involve all firms coordinating on a single platform. Often, especially when firms compete for consumers or on other levels, there may be a "congestion" effect that reduces $U_i(\cdot)$ after a certain number of firms join a given platform—i.e., the *inter-* versus *intra-* platform discussed earlier. In this case, I can assume a fairly unrestrictive condition in that $U_B(\cdot)$ is greater than $U_A(\cdot)$ for low x, and $U_A(\cdot)$ is greater than $U_B(\cdot)$ for higher x.

Assumption 3.2. *("Single Crossing"): There exists $\hat{x}$ such that $U_A(x) < U_B(x)\ \forall\ x < \hat{x}$ and $U_A(x) \geq U_B(x)\ \forall\ x \geq \hat{x}$.*

If either of these conditions hold, then the following proposition shows that complete market tipping must occur in any equilibrium.

Proposition 3.4. *Assume A.3.1. Then there is no interior equilibrium, and there exists an equilibrium where all N firms join platform A iff*

$$\frac{1}{N}V_A(N) + U_A(N) \geq \frac{1}{N}V_B(0) + U_B(0)\ , \tag{3.6}$$

else all firms joining B is an equilibrium.

Assume A.3.2. Then there is no interior equilibrium, and there exists an equilibrium where all N firms join platform A iff

$$\frac{1}{N}V_A(N) + \max_x(U_A(x)) \geq \frac{1}{N}V_B(0) + \max_{x'}(U_B(x'))\ . \tag{3.7}$$

It is easy to construct examples which satisfy the conditions of Proposition 3.3 as well as either A.3.1 or A.3.2. In these cases, Proposition 3.4 clearly emphasizes the importance of contingent transfers in sustaining interior equilibrium. Again, note that efficiency may also be precluded here as neither A.3.1 nor A.3.2 includes platform profits.

3.3 Continuum of Firms

The analysis in this paper is primarily meant to analyze platform competition for a finite number of oligopolistic firms. As N grows large, the marginal contribution of an individual firm to platform payoffs $(V_i(N) - V_i(N-1))$ will typically decline; as noted before, this tends to support more allocations (both interior and corner) as equilibria. I show here that this intuition carries over when there is a continuum of firms.

Assume now there is a measure N of firms, where each firm is atomless. If measure x of firms join platform A, let $F_i(x) \equiv U_i(x) + t_i(x)$ and $G_i(x) \equiv V_i(x) - \eta_i t_i(x)$ be defined as before, where η_i now represents the measure of firms joining platform i. The following proposition states that if platforms are able to offer contingent transfers to each firm based on the measure of firms that join each platform, *all* potential allocations are sustainable in equilibrium; however, with non-contingent transfers, complete market tipping will occur under similar assumptions as before.

Proposition 3.5. *If platforms are able to offer contingent transfers, any $x \in [0, N]$ joining platform A can be sustained as an equilibrium. If platforms can offer only non-contingent transfers and either*

A.3.1 or A.3.2 holds, there is no interior equilibrium and only complete market-tipping equilibria exist.

4 Example: Marketplace

This section applies the model and results to a marketplace which must first attract merchants to its site; the marketplace's profits are dependent on advertising, which in turn is a function of the number of consumers that visit.

I return to the setting with N oligopolistic firms. Each firm produces differentiated goods, and needs to locate in either marketplace A and B in order to sell to consumers. These platforms are located at the endpoints of the unit interval. There is an atomless mass of consumers of measure 1 evenly distributed along the interval, with each consumer's location is given by a type $\theta \in [0, 1]$. Consumers can only visit one market, where they spend \$1 of their income. A consumer of type θ who visits market A receives utility

$$u_A(\theta) = \bar{u}_A + \omega_A(x) - \tau\theta$$

where $\bar{u}_A$ is the base utility from visiting platform A, $\omega_A(x)$ is the utility from shopping at marketplace A given there are x firms located there, and τ is the travel cost to the marketplace. Similarly,

$$u_B(\theta) = \bar{u}_B + \omega_B(x) - \tau(1 - \theta)$$

Assume $\bar{u}_A = \bar{u}_B$, and they are both high enough such that the entire segment is always served for all x. Thus total demand for each market place will be

$$D^A(x) = \frac{\omega_A(x) - \omega_B(x)}{2\tau} + \frac{1}{2} \qquad\qquad D^B(x) = \frac{\omega_B(x) - \omega_A(x)}{2\tau} + \frac{1}{2}$$

Marketplaces do not charge consumers for visiting, but may gain revenues from advertising or incur costs from serving consumers, both of which will be proportional to demand. Let $V_i(x) = \gamma(D^i(x))D^i(x)$ for platform i, where $\gamma_i(D^i(x)) > 0$ represents the per-customer rate that platform i receives from advertising revenues.

For expositional and analytical simplicity, assume that firms set prices after consumers visit markets, thereby separating pricing decisions from marketplace demand effects.[16] Thus, first firms choose which marketplaces to join, then consumers visit a marketplace, and then firms set prices for their goods.

Consumers have CES preferences over goods offered in a marketplace; their utility for visiting A is given by $\omega_A(x) = (\sum_{j\in A}(q_j)^\rho)^{1/\rho} - \sum_{j\in A} q_j p_j$ whereby the number of firms in marketplace A is given by x, q_j is the quantity of good j consumed by an agent, p_j is the price charged by firm j, and $\rho \in (0, 1)$ is the degree of product differentiation. Each consumer who visits marketplace A

[16] None of the results in this example will change if pricing took place before consumers chose.

will spend share $q_j(x) = (p_j^{(-1)/(1-\rho)}/(\sum_{k\in A} p_k^{(-1)/(1-\rho)})$ of her income on good j.

In a symmetric equilibrium when firms have marginal costs c per unit sold, firms in marketplace A will charge $\hat{p}_i^A(x) = (cx(2-\rho) - c))/(x-1)$ and the total quantity sold by each firm will be $\hat{Q}_i^A(x) = (D^A(x))/x$. (In the case that $x = 1$ and there is only one firm in the marketplace, that firm will charge $p = \$1$ and capture all of a consumer's surplus.) Thus

$$\omega_A(x) = (x(\frac{1}{x\hat{p}_i^A(x)})^\rho)^{\frac{1}{\rho}} - 1$$

The value functions for both firms and platforms contingent on x firms joining platform i can thus be computed from the following equations: $U_i(x) = \hat{p}_j^i(x)\hat{Q}_j^i(x)$ and $V_i(x) = \gamma(D^i(x))D^i(x)$.

For a fixed c and γ, it is straightforward to show the following:

- **Platform Differentiation**: $\exists\underline{\tau}$ such that for all $\tau \geq \underline{\tau}$, $x = N$ cannot be an equilibrium; for all $\tau < \underline{\tau}$, $x = N$ is sustainable. As transportation costs decrease, it is akin to the marketplaces becoming less horizontally differentiated, which makes it easier to sustain a market tipping equilibrium. The greater the transportation costs, the more likely it is that certain consumers would be better served by a marketplace with a single firm nearby than a marketplace with all the firms located at the opposite end of the space. Even a consumer attending a market with just a single firm means that she would get 0 utility from purchasing goods, she still derives utility $\bar{u}_i$ from visiting the marketplace. As τ increases, more and more consumers would rather visit the single firm market, and by Proposition 3.3, this would mean that a marketplace with just one firm derives enough surplus to prevent complete tipping.

- **Network Effects** For a fixed $\tau > 0$, $\exists\underline{\rho} \in (0,1)$ such that for all $\rho < \underline{\rho}$, $x = N$ is sustainable in equilibrium. Lowering ρ is equivalent to increasing the returns in a consumer's utility to product differentiation; it also can be seen as the strength of cross market *network effects*—consumers increasingly value having more firms located in the marketplace they visit. Thus, as ρ falls, the LHS of equation (3.1) rises and the RHS falls, thereby allowing a single dominant platform to be stable. Thus, as network effects are more influential in informing a consumer's choice, market tipping becomes more likely.

- **Platform Saturation** If γ is constant, it has no effect on whether or not an outcome is an equilibrium since it does not affect the ratios $(V_A(x) - V_A(x+1))/(V_A(x))$ and $(V_B(x) - V_B(x-1))/(V_B(x))$ for any x. However, if these ratios fall sharply $(\partial\gamma(D^i(x))/\partial x \ll 0)$, then the marginal contribution of an additional firm to either platform falls; by Proposition 3.3, this makes it more likely for market splitting equilibria to be stable.

5 Concluding Remarks

This paper has provided one approach for analyzing the competition between multiple platforms to get an oligopolistic group of firms "onboard." This contracting game and the associated restrictions on the contracting space (i.e., transfers between certain sets of agents are prohibited) can help explain whether or not a market will tip or sustain multiple platforms, and has dramatic effects on the efficiency of the equilibrium outcome as well as the division of surplus between players. In these examples, it is clear how expanding the contracting space to include contingent contracts may actually *worsen* efficiency, as they help sustain interior market-splitting equilibria when market-tipping may be efficient.

As illustrated in the marketplace example from the previous section, the model provides predictions that are consistent with intuition: factors such as greater platform differentiation, firm market power, weaker network effects, and contingent contracts contribute to the existence of multiple platform markets; again, these equilibria may be potentially inefficient.

I conclude with two final remarks. First, it is worth noting that in contexts where firms and platforms are not the only players in the market, the pricing decision of a platform—which side of the market to subsidize and which side of the market to charge—is a topic that has received considerable attention when both sides of the market are non-strategic, but very little when one side is oligopolistic. In the framework of this paper, prices that will be charged by both platforms and firms influences the underlying primitives of the bilateral contracting game, and thus can be used, e.g., as a means of committing players to a particular bargaining position. Secondly, although the analysis has shed light on when a market may tip or sustain multiple platforms, it has done so by taking the underlying profits realized by agents as given and restricting firms to only single-home. Extensions along these dimensions remain the subject of future work.[17]

A Appendix: Proofs

Proof of Proposition 2.1. Recall $F_i(x) = U_i(x) + t_i(x)$ is the payoff to a firm for joining platform i given x firms join platform A. Since F_i is real valued with a compact domain, let $F_A(\cdot)$ attain its maximum value at $x_A^{\max}$ and $F_B(\cdot)$ attains its maximum value at $x_B^{\max}$. If there are multiple points at which F_i attains a maximum, let $x_A^{\max}$ be the greatest x and let $x_B^{\max}$ be the smallest x such that the respective functions are maximized.

Assume for now that $F_A(x_A^{\max}) \geq F_B(x_B^{\max})$. Note that any $x' < x_A^{\max}$ cannot be an equilibrium: if a coalition of $x_A^{\max} - x'$ firms switch from platform B to platform A, they will receive $F_A(x_A^{\max})$ which is strictly preferred to $F_B(x')$ (even if $F_B(x_B^{\max}) = F_A(x_A^{\max})$ and $x_B^{\max} < x_A^{\max}$, by A2.3, this preference relation still holds). This deviation is self-enforcing since no subcoalition of the $x_A^{\max} - x'$ firms will wish to deviate back to platform B, since $F_A(x_A^{\max})$ is again strictly preferred to $F_B(x')$ $\forall$ $x' < x_A^{\max}$. Thus, any potential equilibrium $\hat{x} \geq x_A^{\max}$. Furthermore, any possible deviation from $x' > x_A^{\max}$ to an $x'' < x_A^{\max}$ is not self-enforcing, since there is subcoalition that can self-enforceably deviate back to $x_A^{\max}$.

Using this logic, the proof proceeds by induction. Assume that at iteration n, the only range of possible equilibria are contained within $\Delta^n \equiv \{\underline{x}^n, ..., \overline{x}^n\} \subset \Delta^{n-1}$, where $F_A(\underline{x}^n)$ is strictly preferred to $F_B(x')$ $\forall$ $x' \in \{\underline{x}^{n-1}+1, ..., \overline{x}^{n-1}-1\}$ and $F_B(\overline{x}^n)$ is strictly preferred to $F_A(x'')$ $\forall$ $x'' \in$

[17] For some examples of progress along these lines, Hagiu and Lee (2011) provide a model of bargaining between two platforms and a single firm when the firm is allowed to multihome; Lee (2012) provides a structural model to empirically estimate the underlying payoff functions for platforms and firms (conditional on the entire network structure) where again firms may support multiple platforms; and Lee and Fong (2012) provides a model of contracting and bargaining in general networked environments, where the number of agents on each side of a market is unrestricted.

$\{\underline{x}^{n-1}+1, \overline{x}^{n-1}-1\}$. Observe that if these conditions hold, any possible deviation from $x \in \Delta^n$ to $x' \notin \Delta^n$ cannot be self-enforcing—there would be an $m \leq n$ and either an $\underline{x}^m$ or $\overline{x}^m$ such that a self-enforcing subsequent deviation would exist. Let $x_A^{\max,n}$ be the maximum value of $F_A(\cdot)$ attained on Δ^n (not including $\underline{x}^n$), and let $x_B^{\max,n}$ be the maximum value of $F_B(\cdot)$ (not including $\overline{x}^n$). If $F_A(x_A^{\max,n})$ is preferred by firms to $F_B(x_B^{\max,n})$, then let $\underline{x}^{n+1} = x_A^{\max,n}$ and $\overline{x}^{n+1} = \overline{x}^n$. Otherwise, let $\underline{x}^{n+1} = \underline{x}^n$ and $\overline{x}^{n+1} = x_B^{\max,n}$. It is trivial to see that the conditions of the inductive argument are satisfied by $\underline{x}^{n+1}$ and $\overline{x}^{n+1}$.

Because N is finite, this process ends in a finite number of steps, which occurs when $\underline{x}^n = \overline{x}^n$ for some terminal n. Denote this point $\hat{x}$. By construction, $\hat{x}$ will have no self-enforcing coalitional deviations, and consequently be the unique CPNE of this second stage game. Allowing $\underline{x}^1 = x_A^{\max}$ and $\overline{x}^1 = N$ completes the proof.

If $F_A(x_A^{\max}) < F_B(x_B^{\max})$, then the proof follows as above except $\underline{x}^1 = 0$ and $\overline{x}^1 = x_B^{\max}$. □

Proof of Proposition 3.2. Before proving the proposition, I state the following lemma:

Lemma A.1. *Given t_B, A can induce any outcome $\tilde{x} > 0$ only by offering:*

$$t_A(\tilde{x}) \geq U_B(\tilde{x}-1) + t_B(\tilde{x}-1) - U_A(\tilde{x}) \quad (A.1)$$

$$t_A(\tilde{x}-1) \geq \max_{x<\tilde{x}-1} U_B(x) + t_B(x) - U_A(\tilde{x}-1) \quad (A.2)$$

$$t_A(x) < U_B(\tilde{x}-1) + t_B(\tilde{x}-1) - U_A(x) \text{ for all } x > \tilde{x} \quad (A.3)$$

Furthermore, given t_A, B can induce any outcome $\tilde{x}$ with a similarly defined set of transfers:

$$t_B(\tilde{x}) > U_A(\tilde{x}+1) + t_A(\tilde{x}+1) - U_B(\tilde{x})$$

$$t_B(\tilde{x}+1) > \max_{x>\tilde{x}+1} U_A(x) + t_A(x) - U_B(\tilde{x}+1)$$

$$t_B(x) < U_A(\tilde{x}+1) + t_A(\tilde{x}+1) - U_B(x) \text{ for all } x < \tilde{x}$$

The lemma is easy to prove. Take the first case, where t_B is given. Note that if t_A is as defined in equations (A.1)–(A.3), $\tilde{x}$ is in fact a CPNE for the firms. A cannot set $t_A(\tilde{x})$ any lower that in (A.1) or else $\tilde{x}$ would not be stable and any single firm on A would wish to join B instead. Furthermore, as long as $t_A(\tilde{x}-1)$ is set as high as it is in (A.2), any coalitional deviation of firms from A to B is not self-enforcing. Finally, by (A.3), no more firms would wish to join A than $\tilde{x}$ since then firms would rather prefer to be on B.

Now to prove the proposition: ($\Rightarrow$) For necessity, assume that equation (3.1) does not hold and $\frac{1}{N}V_A(N) + U_A(N) < U_B(N-1) + V_B(N-1)$, but there exists an equilibrium $\{t_A^*, t_B^*, x^* = N\}$. It must be that $V_A(N) - Nt_A^*(N) \geq 0$, otherwise A would be receiving negative payoff and could profitably deviate to receiving 0 by demanding sufficiently high transfers for all realizations of x causing no one to join A. So $t_A^*(N) \leq \frac{1}{N}V_A(N)$. However, this implies that B could offer $t'_B(N-1) = t_A^*(N) + U_A(N) - U_B(N-1) + \epsilon$ (and demand sufficiently high transfers for other network realizations), steal one firm away, and end up with positive (non-zero) payoffs since $V_B(N-1) - t'_B(N-1) \geq V_B(N-1) + U_B(N-1) - (\frac{1}{N}V_A(N) - U_A(N)) > 0$. This means there could not have been an equilibrium where all N firms joined A, and there is a contradiction.

($\Leftarrow$) For sufficiency, the proof is constructive. Assume equation (3.1) holds.

Let

$$\begin{aligned}
t_A^*(N) &= U_B(N-1) + V_B(N-1) - U_A(N) \ , \\
t_B^*(N) &= 0 \ , \\
t_B^*(N-1) &= V_B(N-1) \ , \\
t_B^*(x) &> \max_{x' < N-1} \left(\frac{1}{x'+1}(V_A(x'+1) - V_A(N)) + \frac{N}{x'+1} t_A^*(N) - U_B(x') + U_A(x'+1) \right) \ , \\
& \qquad \forall\, x < N-1 \ , \\
t_A^*(N-1) &= \max_{x < N-1} U_B(x) + t_B^*(x) \ , \\
t_A^*(x) &= \frac{V_B(x)}{N-x+1} + U_B(x-1) - U_A(x) \ \forall\, x < N-1 \ .
\end{aligned}$$

Note that in this case, all firms will select A in a CPNE by virtue of the fact that $t_A^*(N-1)$ is set high enough to preclude any self-enforcing deviation from $x^* = N$. A receives positive profits (since $V_A(N) - N t_A^*(N) > 0$ by equation (3.1) and A2.1), cannot reduce its transfers without losing a firm to B (by construction of $V_B(N-1)$), and cannot induce another outcome x and do better (which follows from the construction of $t_B(x)$ and lemma A.1). Finally, B cannot change its transfers in order to get any firms to join without incurring negative payoffs since it must offer enough to overcome transfers $t_A(x)$. Thus, I have constructed an equilibrium where the outcome is N. □

Proof of Proposition 3.3. Proving necessity is straightforward: $t_A^*(x^*)$ needs to be as large as the value on the RHS of equation (3.2), or else platform B could profitably deviate and induce outcome $x^* - 1$ by offering a transfer schedule as described in lemma A.1. Similarly, $t_B^*(x^*)$ needs to be as large as the RHS of equation (3.3), or else platform A could profitably deviate by inducing outcome $x^* + 1$. However, if either equation (3.4) and (3.5) did not hold, then a platform would be earning negative profits and would do strictly better by not having anyone join at all. So in order for there to exist transfers that induce x^* in equilibrium, the conditions in the proposition must hold.

For sufficiency, construct transfers as follows: let $t_A^*(x^*)$ and $t_B^*(x^*)$ be as in equations (3.2) and (3.3). Set

$$\begin{aligned}
t_A^*(x^*+1) &= U_B(x^*) + t_B(x^*) - U_A(x^*+1) - \epsilon \ , \\
t_B^*(x^*-1) &= U_A(x^*) + t_A(x^*) - U_B(x^*-1) \ .
\end{aligned}$$

so that each platform cannot reduce their transfers offered for x^* without causing firms to switch their platform choice.

Let $K = \max\{\max_x V_A(x), \max_x V_B(x)\}$. Define the remaining transfers as follows:

$$t_A^*(x) = \begin{cases} \max\{K, 2K\} - U_A(x) & \text{if } x < x^* \\ \min\{K, 2K\} - U_A(x) & \text{if } x > x^* + 1 \end{cases}$$

$$t_B^*(x) = \begin{cases} \min\{K, 2K\} - U_B(x) & \text{if } x < x^* - 1 \\ \max\{K, 2K\} - U_B(x) & \text{if } x > x^* \end{cases}$$

It is easy to show that given these transfers: (i) firms will choose x^* in a CPNE; (ii) no platform can reduce the amount paid for the outcome x^*; (iii) no platform can induce another outcome $\tilde{x} \neq x^*$ and receive a higher payoff; (iv) platforms receive positive profits. Thus, given the conditions of the proposition, there exists an equilibrium where x^* firms join platform A. □

Proof of Proposition 3.4. With non-contingent offers, each platform can only make a single transfer offer t_i for affiliation. If A.3.1 holds, then it is straightforward to see that regardless of the values of $\{t_A, t_B\}$, no interior equilibrium is a CPNE for the firms. Furthermore, $t_A = \frac{1}{N}V_B(0) + U_B(0) - U_A(N)$ and $t_B = \frac{1}{N}V_B(0)$ comprise equilibrium transfers as long as equation 3.6 holds: A makes positive profits, B does not find it profitable to increase its own transfers, and all firms choose to join platform A.

If assumption 3.2 holds, then again it is straightforward to see that no interior equilibrium is a CPNE for the firms, regardless of $\{t_A, t_B\}$: if $\arg\max_i[\max_x[U_i(x) + t_i]] = A$, then all firms will join A; otherwise, all firms will join B. The necessity and sufficiency of equation 3.7 follows from the same steps as the previous proofs. □

Proof of Proposition 3.5. Contingent Transfers. Let $K = \max\{\max_x(V_A(x)/x), \max_x(V_B(x)/x)\}$. For any $x^* \in [0, N]$, let:

$$t_A^*(x^*) = U_B(x^*) + t_B(x^*) - U_A(x^*) \ ,$$

$$t_B^*(x^*) = U_A(x^*) + t_A(x^*) - U_B(x^*) \ ,$$

$$t_A^*(x) = \begin{cases} \max\{K, 2K\} - U_A(x) & \text{if } x < x^* \\ \min\{K, 2K\} - U_A(x) & \text{if } x > x^* \end{cases} ,$$

$$t_B^*(x) = \begin{cases} \min\{K, 2K\} - U_B(x) & \text{if } x < x^* \\ \max\{K, 2K\} - U_B(x) & \text{if } x > x^* \end{cases} .$$

Given these transfers, (i) no firm wishes to unilaterally deviate from x^* (which, since they are atomless, results in no change in allocation), and (ii) there is no self-enforcing deviation of any firm coalition with positive measure. Finally, $t_A(x^*), t_B(x^*)$ can be set low enough to ensure that platforms both receive positive profits at x^*, and no platform can unilaterally receive higher payoffs by offering different transfers. Thus, x^* can be sustained in equilibrium.

Non-contingent Transfers. The proof and conditions are identical to Proposition 3.4, except equilibrium transfers in the first case (under A.3.1 and (3.6)) are $t_A = V_B(0) + U_B(0) - U_A(N)$ and $t_B = V_B(0)$. □

B Feasible Transfers

In the analysis, platforms could only utilize symmetric, contingent transfers, and also could potentially offer transfers for off-equilibrium realizations that would yield negative payoffs. This section explores how restricting transfers to be "feasible" alters the analysis.

Definition B.1. *A transfer schedule t_i is* feasible *for platform i iff $\eta_i t_i(x) \leq V_i(x) \ \forall \ x \in \{1, ..., N\}$.*

Feasibility implies that for any outcome x, a platform may not promise making total transfers that would exceed $V_i(x)$. The set of feasible transfers for platform A is given by:

$$T_A^f \equiv \{0, (-\infty, V_A(1)), (-\infty, \frac{V_A(2)}{2}), ..., (-\infty, \frac{V_A(N)}{N})\}$$

(and defined similarly for T_B^f). This restriction is akin to (i) assuming platforms do not have access to external financing and (ii) ensuring that all transfer promises are credible.

The issue with imposing feasible transfers is that it can simultaneously make it less and more difficult to maintain a previous equilibrium outcome x:

- for a given t_{-i}, it may be impossible for platform i to induce any outcome x—platforms may not be able to offer sufficiently high transfers in non-realized market structures, something that was crucial in proving propositions 3.2 and 3.3. This may reduce the number of potential deviations to consider for any platform i.
- At the same time, platforms may also be unable to counter other (feasible) deviations from other platforms by not being able to offer its own (infeasible) off-equilibrium transfers.

As a consequence, analysis becomes more complicated and subject to particular functional forms. What can be shown is that for a reasonable restriction on the primitives V_i and U_i, the conditions for complete market tipping can be characterized.

Assumption B.1. *Assume $\frac{1}{x}V_A(x)$ and $U_A(x)$ are monotonically increasing in x, and $\frac{1}{N-x}V_B(x)$ and $U_B(x)$ are monotonically decreasing in x.*

Proposition B.1. *Given AB.1, $x^* = N$ is supportable in equilibrium iff*

$$V_A(N) + NU_A(N) \geq \max_{x<N} \left\{ V_A(x) + \frac{N-x}{N} V_B(0) + (N-x)U_B(0) \right\} \tag{B.1}$$

Proof of Proposition B.1. For necessity, assume that (B.1) does not hold. Note that if N is to be an equilibrium, then it must be that

$$\begin{aligned} t_A^*(N) + U_A(N) &= \max_{x<N} \{ \frac{1}{N-x} V_B(x) + U_B(x) \} \ \forall \ x < N \\ &= \frac{1}{N} V_B(0) + U_B(0) \qquad \text{by AB.1} \end{aligned} \tag{B.2}$$

or else platform B could offer a transfer for some $x < N$ that would cause $(N-x)$ firms to join B and give B positive profits, or A could lower its transfers paid out without losing any firms. At the same time, it must be that

$$\begin{aligned} V_A(N) - Nt_A^*(N) &\geq V_A(\tilde{x}) - \tilde{x}[\max_{x<\tilde{x}} U_B(x) + \frac{1}{N-x} V_B(x)] \ \forall \ \tilde{x} < N \\ &\geq V_A(\tilde{x}) - \tilde{x}[\frac{1}{N} V_B(0) + U_B(0)] \ \forall \ \tilde{x} < N \end{aligned} \tag{B.3}$$

or else platform A could changes its transfers to induce an outcome $\tilde{x}$ that would give it higher profits. Substituting (B.2) into (B.3) for $t_A^*(N)$:

$$V_A(N) + NU_A(N) \geq V_A(\tilde{x}) + \frac{N-\tilde{x}}{N} V_B(0) + (N-\tilde{x})U_B(0) \ \forall \ \tilde{x} < N$$

which is (B.1), and there is a contradiction.

For sufficiency, let

$$\begin{aligned} t_B^*(x) &= \frac{1}{N-x} V_B(x) \ \forall \ x < N \\ t_A^*(N) &= \frac{1}{N} V_B(0) + U_B(0) - U_A(N) \\ t_A^*(x) &\leq \frac{1}{x} V_A(x) \ \forall \ x < N \end{aligned}$$

It is easy to show that (B.1) allows $t^*_A(N)$ to be feasible, and that N will be an equilibrium: N firms will join A in a CPNE, and neither A or B can change their transfers and be better off. □

This condition is much stronger than that in Proposition 3.2—the RHS of equation (B.1) is greater than (3.1) (from AB.1). Also, note the presence of $V_A(x)$ on the RHS of (B.1)—whereas before B could make it too costly for A to consider inducing a different market outcome $x < N$, now with feasible transfers this may not be possible; as a result it becomes more difficult for complete market tipping to sustain since A now has a greater possibility of finding a profitable deviation. Consequently, if market tipping is indeed efficient, then this restriction on the transfer space makes it more difficult to achieve efficiency.

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