Do these records covary (1)
Do these records covary (2)
Do these records covary (3)
Do these records covary (4)
Time uncertainty is ubiquitous in geologic, paleoclimate, and other records; and there is a need to objectively test for the relationship between such data.
Testing relationships between time-uncertain series: application to millennial variability during the last glacial

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Outline

1. Background.

2. Analytical derivation of the distribution of maximum covariance between pairs of records that have relative time uncertainty; termed MCTEST (Maximum Covariance between Time uncertain Series Test).

3. MCTEST of the relationship between Dongge Cave $\delta^{18}O$ and atmospheric $\Delta^{14}C$.


5. MCTEST of 22 records that resolve millennial scale variability.
A brief background on testing for covariance between time-uncertain records

- *Clark and Thompson* [1979] used a stretching function to align time-uncertain series.

- *Clark* [1989] designed a test using repeatedly randomly repartitioned time-series but employed a null hypothesis that the sequences were in fact equivalent, making it difficult to evaluate the significance of a match between records that only partially covary.

- *Gordon and Buckland* [1996] generated random series through permutations of the records and employed a null hypothesis of no covariance between records, thus permitting testing of partial relationships between records but not preserving the autocorrelation of the records—a feature that we find to be critical.

- Many other tests have been presented within the fields of speech recognition, logistics, and gene sequencing, but none appear directly applicable.
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Simplest possible case: matching a point against a random time series

\[ P(X(3) \leq c \text{ and } X(4) \leq c \text{ and } X(5) \leq c) = \int_{-\infty}^{c} \int_{-\infty}^{c} \int_{-\infty}^{c} \frac{1}{(2\pi)^{N/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(\nu-\mu)^T\Sigma^{-1}(\nu-\mu)} \, d\nu, \]
The cumulative distribution of maximum covariance is a cumulative normal raised to the number of permissible time realizations,

\[ P(x) = F(x)^m \]

\( F(x) \) is the cumulative normal distribution,

\( m \) is the number of permissible time realizations.
Simplest possible case: more choices, less significance.
Simplest possible case: more distinct choices, less significance.
Next simplest case: matching two short time series.
Arbitrarily complex: can still be represented in this framework.

However, the number of dimensions to integrate over scales as $m^n$, $m$ is the number of values each age control point can take on, $n$ is the number of age control points.
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5. **MCTEST of 22 records that resolve millennial scale variability.**
Do Dongge Cave $\delta^{18}O$ and atmospheric $\Delta^{14}C$ covary?

(from Wang et al., 2005)
Monte Carlo and analytical estimates of the 95% critical value at which the null hypothesis of no covariance between the two records can be rejected are consistent.

Once at least $10^5$ of the $10^{30}$ plausible time model realizations are considered, the 95% critical value exceeds a cross-correlation of 0.3, indicating that the null hypothesis of no covariance cannot be rejected.
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Dynamic time warping, showing the distance matrix
Dynamic time warping, showing the cumulative distance matrix.
Dynamic time warping distance matrix, now with a relative time constraint.
Dynamic time warping cumulative distance matrix, with a relative time constraint.
MCTEST using dynamic time warping

1. Determine the maximum covariance between the adjustable and target time series of interest.

2. Repeatedly randomize the target signal, while preserving its auto-covariance structure.

3. Build up the probability distribution associated with null hypothesis by computing the maximum covariance between the randomized target signal and the actual adjustable time series.

4. Compare the actual maximum covariance against the null distribution.
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5. **MCTEST of 22 records that resolve millennial scale variability.**
A collection of 22 paleoclimate records resolving millennial scale variability during stage 3. (compiled by Clark, P.U. et al., 2007)
Significance of the maximum covariance between GISP2 $\delta^{18}O$ and each of the other records
The significance of the covariance between every pair of records
Summary

1. Analytical and Monte Carlo methods are offered to test whether time-uncertain series covary. Use of dynamic time warping greatly increases the efficiency of these MCTESTs.

2. An MCTEST of Dongge Cave $\delta^{18}$O and atmospheric $\Delta^{14}$C records indicates insignificant covariance.

3. Application to 22 records resolving millennial scale variability indicates broadly significant relationships ($p<0.05$) between Northern Hemisphere and high southern latitude records indicative of the atmospheric state.

4. Possible extensions to MCTEST include accounting for systematic errors in time estimates, physical time lags between records, nonlinear relationships between records, and non-normal data distributions.