Localization of mountain glacier termini in Landsat multi-spectral images

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\begin{abstract}
This paper addresses the quantification of glacier retreat through remote sensing. Specifically, we use multi-spectral Landsat satellite images for the estimation of glacier termini locations. Different frequency bands—including visual, infrared, thermal, and processed bands—are examined with respect to their utility in identifying the location of glacier termini and the associated standard error across several scenes. The methodology is to extract an intensity profile along the glacier path from the spatially registered Landsat imagery such that the complexity of the problem is reduced from 2D (image intensity) to 1D (glacier profile intensity). Local polynomial regression is then used to smooth the 1D glacier profile intensity, where the underlying function is assumed to be corrupted with correlated noise. The glacier terminus is then detected by locating an inflection point in the smoothed glacier profile, where a constrained bandwidth selection method is introduced to ensure a single inflection point along the glacier path. Using our method with thermal band B62 and a standard processed band called normalized difference snow index (NDSI) often permits for separating ice from soil but does not lead to a consistent identification of termini location, relative to ground based observations. We therefore introduce a new processed band that combines B62 and NDSI, termed normalized difference thermal snow index (NDTSI). Applying our method along with NDTSI to multiple frames from the Franz Josef, Gorner, Rhone, and Nigardsbreen glaciers indicates an ability to accurately and robustly identify the position of glacier termini, though confirmation of skill awaits application to a larger population of observations.
\end{abstract}

\section{1. Introduction}

There exists a need to better catalog changes in mountain glacier systems, both to more fully assess relationships with climate change (Oerlemans, 2005) and to better monitor and predict changes in water supply (Cullen et al., 2006; Moussavi et al., 2008; Thayyen et al., 2005; Collins, 2008). While the retreat of mountain glaciers has been studied via ground measurements (Moussavi et al., 2008), there is a need to develop automated methods to quantify glacier retreat with a wider scope. A promising means for monitoring glacial changes globally is to make use of available remote sensing information. Among the remote sensing information that is available, the Landsat program provides multi-spectral satellite images for the earth over the past 40 years (Landsat; Landsat.Nasa), constituting a unique multi-decadal resource by which to study temporal variations in the size and shape of glaciers (Landsat.Org; Wulder et al., 2008). Numerous studies have focused on using remote sensing to study single glaciers of interest (Kodde et al., 2007; Höfle et al., 2007; Sidjak and Wheate, 1999), but in order to catalog the temporal changes in even a small percentage of the vast number of glaciers worldwide, a robust and semi-automated methodology would be beneficial, though such an approach is complicated by cloudiness, shadows, seasonal snowfall that can be difficult to distinguish from ice, debris atop glaciers, and other sources of uncertainty.

Here we explore the degree to which Landsat multispectral images can be used to estimate glacier retreat by applying an automated method, where performance is judged against ground measurements (National Snow and Ice Data Center (NSIDC)). In the proposed approach, we first extract an intensity profile along the glacier path for different spectral bands of spatially registered Landsat images. In this way, for each band, we solve a 1D problem by locating the glacier terminus, instead of solving a more complex 2D problem by segmenting the image and classifying the glacier. The glacier path was sketched once for each glacier in this paper and similar results were obtained where different users sketched the glacier path. However, future research will be conducted to detect the glacier path using image processing and pattern recognition methods. The main goal of this paper is to use and compare different Landsat spectral bands in their ability to extract the glacier terminus as the feature of interest. We show that estimation of the glacier terminus through remote sensing is feasible and
promising for purposes of significantly augmenting the limited number of presently available ground based measurements of terminus retreat.

Several image processing steps are needed prior to application of the primary statistical methods presented here. Briefly, the dataset for each glacier is prepared as follows: (1) search the glacier of interest in Landsat multispectral data bank; (2) download the large scenes of interest for each glacier separately; (3) register the multispectral images; The images are automatically registered based on geographical coordinates; (4) crop the glacier area from the large scenes; (5) clean up the dataset by removing the unusable images including those covered by clouds or corrupted by black stripes due to malfunctioning of the Landsat satellite sensors; and (6) sketch the glacier path once for each glacier on spatially registered Landsat images. Note that determining the glacier path, whether it is through manual sketch of the glacier path or through image processing, is presently of secondary interest.

In the proposed method mountain glacier termini locations are detected through identifying inflection points in the smoothed intensity profile along the glacier path. Detection of abrupt changes is a challenging problem in engineering and science. The inflection and change point detection methods can be divided in two groups: (i) on-line methods, to catch a change as soon as it occurs by sequentially inspecting the data with applications in quality control and real time surveillance systems (Desobry et al., 2005; Ke et al., 2007; Mei, 2006); (ii) off-line methods, to detect one or several changes by observing and processing the entire dataset at once with applications in computational biostatistics and off-line computer vision systems (Gustafsson, 1996; James et al., 1987; Takeuchi and Yamashiti, 2006; Yakir et al., 1999; Yao, 1988; Gijbels and Godeaiaux, 2004; Joo and Qiu, 2009). In this paper, we introduce a constrained bandwidth selection in local polynomial regression for inflection point detection under independent or correlated noise. We particularly consider B62, a thermal band in the infrared spectrum, and normalized difference snow index (NDSI), a processed band (Figs. 1 and 2) that measures the reflectance differences of the visible band B20 and the short wave infrared (IR) band B50, defined by Seidel et al. (2004) and Salomonsen and Appelb (2004)

\[ NDSI = \frac{B20 - B50}{B20 + B50} \]

NDSI can usually distinguish ice and soil by combining IR band B50 and visible band B20. The advantage of NDSI for our application is that, in contrast with the other bands, the NDSI glacier images are not as corrupted by shadows as the other bands (Figs. 1 and 2). As depicted in Fig. 1(ii), the glacier is visible in the NDSI image despite the partial loss of contrast due to the shadows.

Therefore, the thermal band B62 and the processed band NDSI were chosen because of their unique characteristics to preserve the visual representation of the glacier in the multispectral Landsat images. While B62 is less affected by shadows, NDSI has higher spatial resolution, and as we will see, one may be better than the other for glacier terminus detection depending on the scene. We should point out that clouds occasionally occlude the glacier terminus in all visual, IR, and processed bands as depicted in Fig. 2 (third column), and that we are unable to make use of such cloudy images.

In order to locate the glacier terminus, a path was manually drawn on the glacier image and applied to all the spatially registered images of the same glacier. Image intensity profiles along the path were extracted from the images across the various bands. The proposed approach for detecting the inflection point along the glacier path intensity profile follows in detail.

2.2. Local polynomial regression for data with independent or correlated noise

The proposed approach applies spatial smoothing of noisy observations while controlling the smoothing bandwidth to ensure a single inflection point under independent or correlated noise. Here, noisy observations of the surface characteristics are made using intensity along the glacier path from multispectral Landsat images. To model the intensity profile along the glacier path, suppose that \( n \) pairs of observations \( (s_i, Y_i) \), \( i = 1, 2, \ldots, n \) consist of a response variable \( Y \) and a location \( s \) on the glacier path and are related by the signal-plus-noise model

\[ Y_i = r(s_i) + \varepsilon_i, \quad \varepsilon_i \sim N(0, \Sigma), \quad i \in [1, n] \]

where \( Y_i, i \in [1, n] \) are observed noisy samples, \( \varepsilon_i \) is correlated Gaussian noise \( N(0, \Sigma) \), and \( r \) is an unknown underlying regression function. The regression function \( r \) can be locally approximated at the point \( s \) by a polynomial of order \( P \) that minimizes the locally weighted mean squared error

\[
    f(A(s)) = \sum_{i=1}^{n} K_i(s_i - s) \left( Y_i - \sum_{p=0}^{P} \hat{a}_p(s) (s_i - s)^p / p! \right)^2 = (Y - XA)^T W (Y - XA) \]

2. Locating the glacier terminus

2.1. The glacier in multi-spectral satellite imagery

Typical multispectral Landsat images are shown in Fig. 1 where false color (RGB = B50, B40, B30) is used to visualize the Franz Josef glacier (blue). Different frequency bands in the multispectral Landsat dataset are investigated to select the best one for our application. The Landsat 7 multispectral datasets contain nine bands including three visual bands (B10, B20, and B30); three infrared bands (B40, B50, and B70); two thermal bands (B61 and B62, see Fig. 1), and one panchromatic band (B80). Bands B10–B50 and B70 have 30 -m resolution, B61 and B62 have 60 m resolution, and B80 has 15 m resolution. Note that Landsat 5 contains only seven bands.

Thermal sensors respond to an object’s temperature at wavelengths of 10.4–12.5 µm and, unlike for other bands, are almost unaffected by shadows (Figs. 1 and 2). For instance, when visible bands B20 and B30 are compared with thermal band B62 for typical images of Franz Josef glacier, the visible bands are corrupted and occluded by shadows from the adjacent mountains, where as the B62 image renders that glacier completely visible.

The proposed approach is applied to several Landsat 5 and Landsat 7 multispectral images of our glaciers: Franz Josef in New Zealand, Nigardsbreen in Norway, and Gorner and Rhone in Switzerland. As an objective metric, we compare the estimated locations of termini by the proposed approach as function of time against ground measurements of terminus retreat. Results show the feasibility of using the proposed method in conjunction with Landsat multispectral imagery for estimation of terminus location changes over time.
Fig. 1. Landsat multispectral images of Franz Josef glacier in New Zealand. (i) A typical Landsat 5 image, where the glacier is clearly recognizable in all spectral bands. (ii) A Landsat 7 image, where the glacier path is partially occluded with shadows in all spectral bands but B62 thermal band. The shadow effect is completely visible in NDSI.
where the polynomial coefficients are estimated by
\[ \hat{A}(s) = (\hat{a}(s), \hat{a}'(s), \hat{a}''(s), \ldots, \hat{a}^{(p)}(s))^T, \]
s is a point in a neighborhood of \( s \),
and \( K_c(s) \) is a kernel with bandwidth \( c \)
\[ K_c(s) = \frac{1}{c} K\left( \frac{s}{c} \right), \quad \int K(s) ds = 1. \tag{4} \]

Minimization of (3) is a least squares problem whose solution is given by
\[ \hat{A}(s) = (X_s^T W X_s)^{-1} X_s^T W Y = LY \tag{5} \]
where \( L = (X_s^T W X_s)^{-1} X_s^T W \) is the smoothing matrix,
\( X_s \) is a \( n \times (p + 1) \) matrix
\[
X_s = \begin{pmatrix}
1 & s_1 - s & \cdots & \frac{(s_1 - s)^p}{p!} \\
1 & s_2 - s & \cdots & \frac{(s_2 - s)^p}{p!} \\
\vdots & \vdots & \ddots & \vdots \\
1 & s_n - s & \cdots & \frac{(s_n - s)^p}{p!}
\end{pmatrix},
\]
\[
\frac{d^3}{ds^3} \text{Se}(D) \approx \text{Se}'(D) \frac{d^3}{ds^3} \text{Se}(D).
\]

Fig. 2. Landsat images of Franz Josef, each column corresponding to a different date, where the glacier path is clear in the left column, is partially occluded with shadows in the middle column, and is occluded by clouds in the right column. (Top row) False color (RGB = B50, B40, B30). (Middle row) NDSI. (Bottom row) B62 (B60 for Landsat 5 depicted in the first column).

Fig. 3. The slope of the second derivative, i.e. the third derivative, at the estimated change point is approximately \( r''(D) \approx \text{Se}'(D)/\text{Se}(D) \).
and \( W_s \) is a \( n \times n \) diagonal matrix

\[
W_s = \begin{pmatrix}
K_r(s_1 - s) & 0 & \cdots & 0 \\
0 & K_r(s_2 - s) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & K_r(s_n - s)
\end{pmatrix}
\]  

(7)

To estimate \( r(s) = a(s) \), the inner product of the first row of \( L \) with \( Y \) is computed by

\[
r(s) = (\{0 \ 0 \ \cdots \ 0\} \times L)Y = e^TLY = \sum_{i=1}^{n} l_i(s)Y_i
\]  

(8)

where \( l_i(s) = e^T L = (l_1(s), l_2(s), \ldots, l_n(s))^T \). The variance of this estimator for correlated noise that is of interest here is

\[
\text{var}(r(s)) = l_i(s) \Sigma_l l_i(s)
\]  

(9)

Note that in the case of independent noise, where \( \Sigma_l = \sigma^2 \cdot I \), we have \( \text{var}(r(s)) = \sigma^2 \|l(s)\|^2 \). Similarly, the derivatives \( r''(s), r'''(s), \ldots, r^{(p)}(s) \) can be estimated by the inner product of \( Y \) with the second row, third row, and \((p + 1)\)th row of \( L \), respectively.

2.3. Termminus detection and bandwidth selection

2.3.1. Inflection point detection

We seek to detect an inflection point \((D)\) on the estimated curve that is negative in the sense that the second derivative \( r''(s) < 0 \) to positive \( r''(s) > 0 \) and the first derivative has a local minimum such that \( r'(s) < 0 \)

\[
D = \left\{ s : \hat{r}_2(s) = 0 \land \hat{r}_3(s) < 0 \right\}
\]  

(10)

In a similar way to (8), the estimated 2nd derivative \((\hat{r}_2(s))\) is

\[
r''(s) = (\{0 \ 0 \ 0 \ \cdots \ 0\} \times L)Y = \sum_{i=1}^{n} l_i^2(s)Y_i
\]  

(11)

and its associated standard error in the presence of correlated noise is computed by

\[
\text{var}(r''(s)) = \text{var}(r_2'(s)) = \sigma^2 \|l(s)\|^2
\]  

(12)

Note that in the case of independent noise \((\Sigma_l = \sigma^2 \cdot I)\), the standard error is computed by \( \text{var}(r''(s)) = \sigma^2 \|l(s)\|^2 \). The positive inflection point can be identified in a similar way where the second derivative changes sign from positive to negative. As illustrated in Fig. 3, the standard error of the estimated inflection point location is estimated by

\[
\text{Se}(D) \approx \frac{\text{Se}(r''(D))}{r''(D)}
\]  

(13)

where \( D \) is the identified inflection point location. Here assuming correlated noise, using (12) the standard error of the second derivative at the inflection point is estimated by

\[
\text{Se}(r''(D)) = \sqrt{r''(D) \Sigma_l r''(D)}
\]  

(14)

and the third derivative \( r''' \) at the inflection point is computed in a similar way as (11) by

\[
r'''(s) = (\{0 \ 0 \ 0 \ \cdots \ 0\} \times L)Y = \sum_{i=1}^{n} l_i^3(s)Y_i
\]  

(15)

2.3.2. Bandwidth selection

Conventionally, the optimal bandwidth is chosen using the leave-one-out cross validation score

\[
\text{CV} = \hat{R}(\gamma) = \frac{1}{n} \sum_{i=1}^{n} (Y_i - \hat{r}_{i-1}(s))^2
\]  

(16)

where \( \hat{r}_{i-1} \) is estimated by excluding the \( i \)th pair \( s_i, Y_i \). The optimal bandwidth chosen by cross validation may produce many inflection points. Applying the selected optimal bandwidth obtained by cross validation, we may identify zero, one, or several inflection points and can be difficult to identify a single, true inflection point associated with the underlying unknown curve.

Instead, we propose a new bandwidth selection method involving application of a constraint to ensure that the smoothed curve \( \hat{r}_f(s) \) has only one inflection point:

\[
\hat{r}_f(s) = \sum_{i=1}^{n} l_i(s)Y_i, \quad |D| = 1
\]  

(17)

Selected bandwidth \( \gamma \) is the smallest bandwidth that satisfies the constraint \(|D| = 1\), where \( D \) is the set of inflection points detected using (10).

The proposed method selects the optimal bandwidth to ensure a single change point. This avoids the uncertainty due to detection of zero or multiple change points when applying standard bandwidth selection methods such as cross validation. Therefore the proposed method increases the accuracy of the estimated change point location for such applications like ours where the true function has only one change point. This method permits a flexible fit to estimate the underlying function by taking advantage of non-parametric regression while avoiding over-smoothing.

2.3.3. Termminus detection

The glacier terminus \((D)\) is where ice stops and gives way to another surface—typically soil, rock, or water. We posit that the terminus can be identified as a point on the intensity profile along the glacier path where the curvature changes sign. Hence, a glacier terminus is located by detecting a single inflection point (17) on the smoothed intensity profile using the method described above. The standard error associated with the estimated terminus location is computed by (13). The actual glacier terminus \( D \) is identified, first, by locating the zero crossing \( D_{0,0} \) of the estimated second derivative \( r''(s) \); and, second, by detecting the local minimum of the estimated first derivative \( \hat{r}'(s) \) in the \( \gamma \) neighborhood of \( D_{0,0} \), i.e. \( D \in (D_{0,0} - \gamma, D_{0,0} + \gamma) \). This approach is discussed in more detail in Section 4.2.

2.4. New index

Relative to NDSI, the B62 band is less affected by shadows but has a lower spatial resolution of 60 m, instead of, 30 m. The visual representation of the glacier is preserved in the thermal band B62 and the processed band NDSI in the multispectral Landsat images; however, one band may be more advantageous for identifying the terminus, depending on the scene. To attempt to capture the qualities of both measures, we introduce a new processed band, called the Normalized Difference Thermal Snow Index (NDTSI) by combining B62 and NDSI:

\[
\text{NDTSI} = \frac{\text{NDSI} - \text{B62}}{\text{NDSI} + \text{B62}}
\]  

(18)

Given that NDSI has a 30 m resolution while the resolution of B62 is 60 m, we interpolated B62 to 30 m resolution in order to compute NDTI and to better permit for inter-comparison of results. Examples illustrating that NDTI can lead to a more accurate and consistent estimate of the terminus are provided below.
Fig. 4. Application of the proposed method to a shadow-free Landsat 5 multispectral image of the Franz Josef glacier on December 30th, 1990. (i)–(ii) Visualization of the glacier using false color (RGB = B50, B40, B30) where estimated terminus locations are marked by a purple disk for B62, a red disk for NDSI, and a yellow disk for the new index NDTSI, and the associated standard errors are marked by correspondingly colored circles. Note that errors are along the path and the circle is only used for visual clarity, and that (ii) is a zoomed-in version of (i). (iii)–(v) Visualization of the glacier using spectral and processed bands where the estimated terminus locations are marked by purple disks for B62 (iii), NDSI (iv), and NDTSI (v). (vi)–(viii) The extracted noisy glacier path intensity profiles (purple), smoothed intensity profiles applying the local polynomial regression by (8) using the proposed constrained bandwidth selection (blue), the point-wise confidence intervals (black) computed using (9), and the estimated terminus locations (yellow disks) obtained by (10) for B62 (vi), NDSI (vii), and NDTSI (viii). (ix)–(x) Estimated first (blue) and second (red) derivatives that are used to locate the glacier terminus for B62 (ix), NDSI (x), and NDTSI (xi). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Fig. 5. Similar to Fig. 4 but for a Landsat 7 multispectral image of the Franz Josef glacier in April 2004 where the glacier path is partially occluded by shadows. Note that overall uncertainties are now larger and that the estimated terminus locations in (i,ii) are the same for NDSI (red) and NDTSI (yellow) but that the latter has a smaller standard error. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
3. Results

The proposed method is applied to estimate glacier termini locations of four glaciers: Franz Josef in New Zealand, Nigardsbreen in Norway, and Gorner and Rhone in Switzerland. We use Landsat 5 and Landsat 7 multispectral images and compare results using the B62 spectral band and the NDSI and NDTSI spectral indices.

3.1. Terminus localization for the Franz Josef glacier

First, the terminus location is estimated along the glacier path in several Landsat 5 and Landsat 7 multispectral scenes of Franz Josef. Fig. 4 shows a shadow-free scene of Franz Josef from December 1990 in false color (RGB where R = B50, G = B40, and B = B30) where the glacier is clearly visible in blue. To estimate the terminus location, intensity profiles are extracted along the glacier path.
using B62, NDSI, and NDTSI. Smoothed intensity profiles are then computed using (8), and the associated point-wise confidence intervals are estimated using (9). A single inflection point has been ensured in the nonparametric regression fit to the intensity data using the constrained bandwidth selection method (17), which is taken as our best estimation of the terminus location. The standard errors associated with the estimated terminus locations are computed using (13).

The estimated terminus location for Franz Josef using B62, NDSI, and the new NDTSI are similar (Fig. 4i and ii) with standard errors of 70 m for B62, 43 m for NDSI, and 47 m for NDTSI, such that each are just slightly larger than their native resolution. It is also instructive to consider a false color scene of Franz Josef in which the glacier path is partially occluded by shadows (Fig. 5). Here, the NDSI image is clearly affected by shadows and leads to noisy observations, whereas the B62 band is much less affected and still

Fig. 7. Similar to Fig. 6 but for a Landsat 5 multispectral image of the Gorner glacier in October 1986 where an ice field affected the glacier terminus on the top left (black). In comparison with Fig. 6, the estimated terminus location using B62 is displaced substantially with respect to NDSI and NDTSI. The large uncertainty in the location given by B62 is explained by the lack of a sharp transition in the intensity profile (vi) as opposed to the other two bands (vii and viii) (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).
provides an obvious inflection point for estimation of the terminus location. The result is that B62 has a much smaller estimated error (17 m) than those estimated by NDSI (438 m) or NDTSI (294 m).

3.2. Terminus localization for the Gorner glacier

A second set of example terminus identification is provided for Gorner to illustrate features of the identification method associated with the presence of seasonal snow or ice. On a cloud free day in August 1985 (Fig. 6) the estimated location of the terminus using all three bands is consistent and the inferred errors are all 26 m or lower. In contrast, ephemeral snow and ice is present in a scene from October 1986 (Fig. 7) that leads to an implausible estimate of the terminus location when using the B62 band, but not when using the NDSI or NDTSI indices. The standard error associated with the terminus location estimated from B62 (331 m) reflects the difficulty in identifying the terminus relative to the standard errors associated with the other bands (both near 30 m). This result illustrates that although B62 can sometimes provide a more accurate detection of the terminus location, such as for the cloudy scene from Franz Josef, it is also liable to major errors, such as for the snowy scene from Gorner. In contrast, the NDTSI offers an apparently more stable compromise between B62 and NDSI and which we expect to result in a more robust estimation of changes in glacier length.

The ability to identify the terminus location using B62 or NDSI may change not only between different glaciers, but for the same glacier over time. Fig. 8 shows the Gorner terminus location estimated along the glacier path intensity profile in nine Landsat 5 and Landsat 7 multispectral scenes between August 1985 and August 2003 using B62, NDSI, and NDTSI. Similar to the foregoing illustrations, the results from each of the three bands are generally comparable under optimal conditions but diverge in the presence of cloud and snow obscuration of the terminus. NDTSI appears to provide a stable compromise between B62 and NDSI among all time points.

3.3. Comparison with ground records

We used Landsat multispectral images to estimate the terminus location in different frequency bands for several scenes of four glaciers taken over past three decades (Fig. 9). As the main concern here is long term changes in terminus position, rather than the short term variations, the estimated terminus location over time was smoothed using a fourth degree polynomial. For validation, the estimated terminus locations using different frequency bands provided an obvious inflection point for estimation of the terminus location. The result is that B62 has a much smaller estimated error (17 m) than those estimated by NDSI (438 m) or NDTSI (294 m).
by the proposed method were compared with the available ground measurements for each glacier. Both ground measurements and estimated locations measure relative change between consecutive time points. Hence to compare them, in each panel of Fig. 9, the mean of the ground measurements $\mu_m$ is shifted to the mean of the estimated locations $\mu_B$ obtained by the band $B$ that has minimum absolute mean difference (min|$\mu_m - \mu_B$|). The ground measurements and estimated locations cannot be compared in absolute terms, but they can be compared in terms of their pattern of change over time. In Fig. 9, the estimated glacier terminus locations using NDSI, NDTSI, and B62 (1st column) have a more similar pattern to the ground measurements than the other frequency bands (2nd and 3rd columns). Among those three, NDSI provides closer pattern to the ground measurement than B62 for Gorner (1st row) and Nigardsbreen (3rd row), while B62 does for Franz Josef (2nd row) and Rhone (4th row). However, our new index, NDTSI, provides a pattern that resembles that of the band that performs better (NDSI or B62) for all four glaciers, performing particularly well for Gorner and Nigardsbreen. The estimated terminus location over time using the visual frequency bands B10, B20, and B30 (2nd column) are generally similar except for Nigardsbreen (3rd row) but none provide a consistent estimate of the ground measurement over time. Among the infrared bands, B30 gives a pattern similar to that of B62 and the ground measurements, whereas B40 does not. We should point out that B50 and B62 are used to compute the processed bands NDSI and NDTSI.

To quantify the difference between the estimated terminus change and the ground measurements, we computed the minimum root mean square error (RMSE). The RMSE is a minimum because, first, the difference of the mean ground measurements ($\mu_m$) and the mean of the estimated locations ($\mu_B$) obtained by the band $B$ is set to zero (|$\mu_m - \mu_B$| = 0) by shifting $\mu_m$ to $\mu_B$. Then, RMSE is computed between the shifted ground measurements and the smoothed estimate of terminus location across the available images for each glacier. Computed RMSE for four glaciers and

<table>
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Fig. 9. Estimated terminus locations over time after temporal smoothing for Gorner (1st row), Franz Josef (2nd row), Nigardsbreen (3rd row), and Rhone (4th row). (Left column) B62 thermal spectral band (blue), NDSI processed band (red), and the proposed processed band NDTSI (purple). (Middle column) Visual spectral bands B10 (red), B20 (green), and B30 (blue). (Right column) Infrared spectral bands B40 (red) and B50 (green), and thermal band B62 (blue). In all panels, ground measurements are depicted in black. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 1

Root mean square error (RMSE) of estimated terminus change using the proposed method vs. ground measurements over time (in meters).
different frequency bands are summarized in Table 1. We observe that NDTSI performs robustly for all studied glaciers and provides smaller or comparable RMSE relative to B62 and NDSI. The thermal band B62 and the infrared band B50 (that is used to compute NDSI) give similar, though generally somewhat larger, values of RMSE.

The estimated terminus locations over time using NDTSI by applying the proposed method before and after temporal smoothing are depicted in Fig. 10 for all four glaciers. There exist short-term variations, possibly related to seasonal and short-term environmental effects, but the long term changes obtained after polynomial smoothing generally capture the pattern found in the ground measurements.

4. Discussion

Comparing the results obtained by application of the proposed method to Franz Josef, Gorner, Rhone, and Nigardsbreen for different spectral bands and the processed band NDSI, it can be concluded that B62 and NDSI produce comparable results for Rhone, B62 is better suited for Franz Josef, and that our approach performs better than using NDSI for Gorner and Nigardsbreen. However, the newly introduced index, NDTSI, generally gives a more accurate location estimate than either B62 or NDSI for all four glaciers in this study. In all cases, the terminus location estimates may be subject to systematic biases, but the ability to recover the pattern of change over time exhibited by the ground measurements shows the promise of using remote sensing for this purpose.

4.1. Contributions

In this paper, mountain glacier terminus detection using Landsat multispectral images is addressed. The contributions can be summarized as follows.

(a) Landsat multispectral satellite images were used to estimate the mountain glacier terminus location. The terminus location was estimated as an inflection point in the glacier path intensity profile extracted from Landsat spectral bands. The results obtained using different frequency bands were compared with the available ground measurement for four glaciers.

(b) A new processed band (NDTSI) computed using B62 and NDSI was introduced for glacier studies via remote sensing. NDTSI was used to estimate the glacier terminus location and was shown to provide a better estimation than any individual Landsat spectral band. NDTSI provides also a more robust estimation in comparison with B62 and NDSI for our application.

(c) A constrained bandwidth selection method in local polynomial regression was introduced for inflection point detection, where the true signal was corrupted with either independent or correlated noise. The standard error associated with the estimated location of the inflection point was computed using the 2nd and the 3rd derivatives of the underlying function.

4.2. Estimation methodology

In many applications including the one introduced in this paper, i.e. glacier terminus detection through multispectral satellite imagery, identification of a single inflection point is important. A nonparametric local regression is applied for curve fitting while the first and the second derivatives of the smoothed curve are employed to detect the glacier terminus. We introduced a constrained bandwidth selection method to ensure a single inflection point under the correlated noise. The smoothed intensity profile was computed using (8) whereas the point-wise confidence interval was estimated by (9). The estimated location of the terminus, i.e. the identified inflection point, was estimated using (10) as the unique zero crossing of the second derivative, where the first and the second derivatives were estimated using $r(s) = \sum_{i} f_i(s)Y_i$ and $r''(s) = \sum_{i} f_i''(s)Y_i$. Theoretically, for a continuous twice differentiable function $r(s)$, the first derivative $r'(s)$ reaches its local optima where the second derivative $r''(s)$ crosses the zero line. However, the estimated second derivative is not equal to the derivative of
the estimated first derivative. Notice in Figs. 4–7 that the estimated first derivative has several local optima while the estimated second derivative has a single zero crossing. As a consequence, the local optima of the estimated first derivative $\hat{r}(s)$ might be shifted to the left or the right of the point where the estimated second derivative $\hat{r}^2(s)$ crosses the zero, by an amount not greater than the bandwidth $\gamma$. For this reason, the glacier terminus $D$ is identified by, first, locating the unique zero crossing $(D_{\hat{r}^2})$ of the estimated second derivative $\hat{r}^2(s)$; and, second, detecting the local minimum of the estimated first derivative $\hat{r}(s)$ in the $\gamma$ neighborhood of $D_{\hat{r}^2}$, i.e. $D \in \{D_{\hat{r}^2} - \gamma; D_{\hat{r}^2} + \gamma\}$.

We have shown that a statistical approach can be effectively applied to detect the glacier terminus through multispectral Landsat images. It has been also demonstrated that thermal band B62 and processed band NDSI have characteristics well suited to the application at hand. Our approach performs better using B62 and NDSI than the other spectral bands that were evaluated. Depending on the glacier and various scenes, however, either the B62 or NDSI may better identify glacier features. Therefore, to preserve a consistent visual representation of the glacier in different scenes for our application, we introduced a new index, NDTSI. NDTSI mimics NDSI in that it increases the contrast by taking the normalized difference of two spectral bands (B20, which has higher intensity for ice, and B50, which has higher intensity for non-ice), but further enhances the contrast by also incorporating the normalized difference of two other measures (NDSI, which has higher intensity for ice, and B62, which has higher intensity for non-ice). In our present application, estimation of terminus location is more accurate and robust using NDTSI because it more sharply identifies the transition between ice and non-ice surfaces.

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References