An Untitled Derivation of the Law of Refraction, December 1681 by G.W. Leibniz[1]

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Let AC be a ray coming from the medium DC into the medium CE, and let the density of the former to the latter be as *d* to *e*. It is asked, how should the ray ACB be directed so that it is the easiest path of all, or that (AC x *d*) + (CB x *e*) is a minimum. Let DC = *l*, and EC = *m*. It is given also that FG = *f*, and let AD = FC = *x*, CG = EB = *f* – *x*. Therefore, AC = $\sqrt{(l^2 + x^2)}$ and CB = $\sqrt{(m^2 + f^2 + x^2 - 2fx)}$.[2] It will then be the case that $d\sqrt{(l^2 + x^2)} + e\sqrt{(m^2 + f^2 + x^2 - 2fx}$ is equal to a minimum.

Therefore, through my method of tangents it is the case that: $(2dx/(\sqrt{(l^2 + x^2)}(AC))) + ((2ex - 2ef)/ (\sqrt{(m^2 + f^2 + x^2 + 2fx)}(BC))) = 0$. That is, it will be the case that AC/BC = dx/e(f-x). Now if we suppose that AC and BC are equal, it will be the case that f – x is to x, as d to e. Therefore, if a circle, with its center at C, is described by the ray CA or CB, AD or "x," – the sine of the angle of incidence – will be to BE or "f – x," – the sine of the angle of refraction – as e, the density of the medium of refraction, will be to d, the density of the medium of incidence, that is, the sines of the angles will be in reciprocal relation to the mediums or densities.

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[1] Translated from the Latin text as found in Ernst Gerland, *Leibnizens Nachgelassene Schriften Physikalischen, Mechanischen und Technischen Inhalts* (Leipzig: B. G. Teubner, 1906; reprinted Ann Arbor, Michigan: University of Michigan University Library, 2006), p. 73. Special thanks to Paul Hoffman, Jeremy Hyman, and Ben King for reviewing the present translation.

[2] Since CB = $\sqrt{(EB)^2 + (EC)^2}$, and EB = f-x.