Why the poor do not expropriate the rich: an old argument in new garb

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Abstract

We consider a political economy with two partisan parties; each party represents a given constituency of voters. If one party (Labour) represents poor voters and the other (Christian Democrats) rich voters, if a redistributive tax policy is the only issue, and if there are no incentive considerations, then in equilibrium the party representing the poor will propose a tax rate of unity. If, however, there are two issues – tax policy and religion, for instance – then this is not generally the case. The analysis shows that, if a simple condition on the distribution of voter preferences holds, then, as the salience of the non-economic issue increases, the tax rate proposed by Labour in equilibrium will fall – possibly even to zero – even though a majority of the population may have an ideal tax rate of unity.

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1. The historical issue and a model preview

The framers of the US constitution extended suffrage only to (male) property-holders because they believed that, were the poor to be given the vote, they would soon expropriate the rich. Property owners, it was believed, would behave ‘responsibly.’ If all citizens have the vote, and median wealth is less than the mean (always true of actual wealth distributions), then a majority of voters (namely,
those whose wealth is less than the mean) should prefer a tax rate of unity, fully redistributing all wealth to the mean.

Nevertheless, twentieth-century universal suffrage has not engendered the expropriation of the rich through the tax system, and a variety of reasons have been offered in explanation, including the following. (1) Voters recognize that there would be adverse dynamic effects to expropriating the rich, who have scarce productive talents which would cease to be supplied were their holders taxed too harshly, and all would consequently suffer (trickle-down); (2) many voters whose wealth lies below the mean entertain the hope that they or their children will someday become richer than the mean, and they shun high tax rates for fear of hurting their future selves or children; (3) even if there would be few dynamic effects from high taxation, as described in (1), the rich convince the citizenry that there would be, with propaganda disseminated through the media, which they control; (4) the citizenry believe that the rich person – and indeed everyone – deserves the wealth he/she receives, and hence high tax rates would be unethical. Marxists have called explanations (3) and (4) instances of ‘false consciousness.’ Putterman (1997) has recently tried to assign degrees of importance to the explanations here suggested, and some others.

In this article, I will propose another possible explanation for the non-expropriation of the rich in democracies, which depends upon there being party competition on a policy space with two dimensions, the first being taxation, the second some non-economic issue, such as slavery/integration, religion, nationalism, or ‘values.’ The proposal I shall offer has nothing to do with incentives and trickle-down: were wealth simply manna from heaven, which fell unequally on the population, the argument I present would still hold.

The model behind the view that those with wealth less than the mean would vote for a tax rate of unity on wealth presupposes that political competition is unidimensional. But, indeed, political competition, in at least the US and Europe, is surely at least two dimensional. Poole and Rosenthal (1991) have shown that roll call votes in the US Congress, going back to 1789, are best explained by a two-dimensional model: knowing the position of congressmen on taxation and race (slavery before the Civil War and integration/civil rights after), one can explain 85% of the variance in roll call votes, and adding a third dimension explains very little more. Laver and Hunt (1992) present empirical evidence that democratic politics are multi-dimensional in a set of over twenty countries. Somewhat more schematically, Kitschelt (1994) argues that, in the main European countries, politics can be understood, in the past thirty years, as being two dimensional, over redistribution and a ‘communitarian’ dimension, whose poles he labels ‘authoritarian’ and ‘libertarian.’ The authoritarian voter wants more police, more defense spending, illegalization of abortion, tough anti-drug legislation, the death penalty (in the US), and is pro-clerical. The libertarian voter wants the respective opposites, and is anti-clerical. Kitschelt argues that the ‘communitarian’ dimension is quite orthogonal to the economic dimension: blue collar workers in manufactur-
ing tend to be redistributionist and authoritarian, while some professional workers are anti-redistributionist and libertarian. On the other hand, many poor, minority voters are redistributionist and libertarian, while the ‘petty bourgeoisie’ are anti-redistributionist and authoritarian. Kalyvas (1996) and Przeworski and Sprague (1986) together form a convincing argument that, in at least the period 1880–1940, both religion and redistribution were important dimensions in European politics.

Suppose, then, that voter preferences are defined over wealth and some non-wealth issue – I will call the second dimension ‘religion,’ for concreteness, from now on. The citizenry’s preferences are characterized by a joint probability distribution over tax-religion policy space. Suppose there are two political parties with policy preferences: one party represents constituents who are poor and anti-clerical (the Labour Party), and the other represents constituents who are rich and pro-clerical (the Christian Democratic Party). It is important to understand that these parties are not Downsiàn – neither wishes to maximize the probability of winning the election per se, but rather to maximize its constituents’ expected welfare. (The distinction between these objectives will become clearer below.)

Given this political institution, here is a very rough intuition for why Labour might not optimize in the electoral contest by proposing a tax rate of unity. Suppose that poor voters are generally anti-clerical, but there is a significant number of pro-clerical poor, and that richer citizens are generally pro-clerical, but there is a significant number of anti-clerical voters among them. Indeed, there may be a substantial number of voters, among the poor, who are so pro-clerical that they will not vote Labour even if Labour proposes a tax rate of unity, as long as the Christian Democrats propose a more pro-clerical policy than Labour. Thus, Labour cannot necessarily win with high probability (more precisely, maximize the expected welfare of its poor, anti-clerical constituents) if it proposes a tax rate of unity, assuming it remains ‘principled’ on the religion issue. It may well be in the interest of Labour’s constituents to propose a tax rate less than one, thereby winning the votes of some richer citizens who are quite anti-clerical. This, Marx might well have said, is an instance of religion’s being the opiate of the masses – i.e., the poor, pro-clerical voters are acting against their ‘real’ interests. (If one thinks of the non-economic issue as race, which is perhaps the most appropriate one for the United States, one might paraphrase Marx by arguing that racism is the opiate of the masses.) But we are not here inquiring into why citizens have these preferences over a non-economic issue. The essential point is that, if voters care deeply about some non-economic issue, and have widely disparate views on that issue, it does not follow that all those whose wealth is less than the mean will necessarily support a party which proposes a tax rate of unity.

To check whether this moderate redistributive policy of Labour can actually be optimal – can be an equilibrium policy in the electoral game – will require a model of party competition between Labour and the Christian Democrats – in particular, a notion of equilibrium in the electoral contest between two parties,
each representing constituents, when the policy space is two-dimensional. The central technical problem facing the analyst is that the natural concept of political equilibrium—a Nash equilibrium in which each party plays a best response to the other’s policy—fails to exist, in pure strategies, with multi-dimensional issue spaces. There are two moves an analyst can usually make in such cases: either to consider mixed strategy equilibria in the one-shot game, or to reconceive of the game as one which takes place in stages, and then use some refinement of perfect Nash equilibrium. The simplest example of the second option is Stackelberg equilibrium in a two-period game.

I do not believe we can reasonably think of parties playing mixed strategies, and so I reject the first option. I find the second option less objectionable, and I pursue it in Section 5 below.

But I believe that even the stage-game tack is a compromise with reality, because it can be argued that parties write their manifestos approximately contemporaneously, and the manifestos determine their platforms. (Indeed, Budge et al. (1993) argue, based on empirical analysis of ten countries, that parties’ platforms adhere closely to their manifestos.) It is therefore advisable to find, if we can, an equilibrium concept which works in the two-dimensional problem in a simultaneous move game between the parties. I introduce such a concept in Section 6—the key is to alter the preferences of the parties from their usual form, based on modelling the intra-party struggle over policy, among its factions. I name such Nash equilibria political unanimity Nash equilibria (PUNE).

My substantive question is: Is there a reasonable condition on the distribution of voter preferences (or traits), such that the equilibrium in the electoral contest between a Labour Party that represents a poor anti-clerical voter, and a Christian Democratic Party that represents a rich, clerical voter, entails Labour’s proposing a tax rate which is significantly less than one?

What I discover, in Section 5 and Section 6, is such a condition, and moreover, that the same condition implies that, whether we model political competition as Stackelberg or as PUNE, the desired result holds. In fact, under either conceptualization of political competition, if the religious issue is sufficiently salient, then the Labour Party will propose a zero tax rate in equilibrium.

In the process of answering the posed question, I will offer an answer to another question as well. Kitschelt has argued that the non-economic dimension (what he calls the ‘communitarian’ issue) has increased in importance in western democracies in the post-war period. Clearly, in a two-dimensional model, as the non-economic issue becomes more salient for voters, we can expect both components of the equilibrium policies to change. Is there any reason to believe that, as the importance of the non-economic issue increases, the equilibrium tax

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1 A number of authors have studied electoral equilibrium between two parties, each of which has policy preferences (or represents constituents) when the policy space is one-dimensional: Wittman (1983); Calvert (1985); Alesina (1988), and Roemer (1994), (1997a), to name several. To my knowledge, there has been no analogous analysis of the two-dimensional model.
policies proposed by the Labour party should decrease, as opposed to increasing, or moving around non-monotonically? We can interpret the main results as answering this question affirmatively, assuming that the key condition on the distribution of voter traits holds.

Section 8 investigates, in a preliminary way, whether the key condition is true of the US and British electorates, where we take the non-economic issue to be, in one case, racial attitudes, and in another, communitarian attitudes. Some tentative predictions about US and British political behavior are drawn from the model.

2. Preliminaries

Here, I present the standard model of competition between partisan parties, applied to our context of a two-dimensional issue space.

Let the space of citizen traits be \( \mathcal{A} = W \times R \), with generic element \((w, a)\), where \( W = [w, \bar{w}] \) is the set of wealth (or income) levels, and \( R \) is the set of religious views, taken to be the real number line. The utility function of a citizen with traits \((w, a)\) over policies \((t, z)\), where \( t \) is a uniform tax rate on wealth or income, and \( z \) is a religious position of the government, is given by \( v(t, z; w, a) \). The population is characterized by a probability distribution on \( \mathcal{A} \). There are two parties: \( ^1 \) Labour, or Left, represents a constituent with traits \((w_L, a_L)\) and the Christian Democratic Party, or Right, represents a constituent with traits \((w_R, a_R)\). Each party, \( i \), proposes a policy pair \( \tau^i = (t^i, z^i) \). We suppose there is a stochastic element in these elections, which I will specify in Section 4, so that, given a pair of policies \((\tau^1, \tau^2)\), there is only a probability that Left (Party 1) will win, denoted \( \pi(\tau^1, \tau^2) \). The function \( \pi \) is known to both parties. Then the pay-off functions of the Left and Right parties are:

\[
\begin{align*}
\Pi^1(\tau^1, \tau^2) &= \pi(\tau^1, \tau^2)v(\tau^1; w_L, a_L) + (1 - \pi(\tau^1, \tau^2))v(\tau^2; w_L, a_L) \\
\Pi^2(\tau^1, \tau^2) &= \pi(\tau^1, \tau^2)v(\tau^1; w_R, a_R) + (1 - \pi(\tau^1, \tau^2))v(\tau^2; w_R, a_R).
\end{align*}
\]

That is, the pay-off of a party at a policy pair is the expected utility of its representative constituent at that pair of policies.

It is generically the case that Nash equilibria in pure strategies, for the game in which the payoff functions are \( \Pi^1 \) and \( \Pi^2 \), do not exist.\(^3\)

\(^3\)I take these parties to be historically given, just as in the Arrow–Debreu model, firms are historically given; I present no analysis which explains how these two particular parties have come to be. I take the parties as representing particular voters, rather than coalitions of voters, as a simplification.

In Roemer (1997a) I prove existence of Nash equilibrium for the one-dimensional electoral game, where parties face uncertainty and represent constituents. Even in that model, conditional payoff functions are not quasi-concave. In the two dimensional model, however, the violation of quasi-concavity is so serious that, generically, pure strategy Nash equilibria do not exist.
3. Expropriation of the wealthy in a unidimensional contest

As a first (easy) exercise with the model of constituency representing parties, I show that, if the policy space is unidimensional (a single tax rate) and taxes are purely redistributive, then a party representing a voter whose wealth is less than the mean will propose a tax rate of unity in Stackelberg equilibrium. Understanding this exercise should help the reader maintain his/her bearings in the more complicated two-dimensional problem to follow. Another reason to study this case is that the analysis differs from that of the Downsian model, where parties have no policy preferences. Most readers will be familiar with the ‘median voter theorem’ of the Downsian model.

Let \( W \) be an interval of real numbers, and let \( g(w) \) be a density on \( W \) characterizing the society’s distribution of wealth. If \( t \) is a proportional tax rate on wealth, then per capita taxes collected will be \( t \int_w g(w) dw = t \mu \), where \( \mu \) is the mean of \( g \). Thus, ‘post-fisc’ wealth of a citizen with wealth \( w \) will be \( (1-t)w + t \mu \). Suppose von Neumann–Morgenstern preferences for wealth are universally risk-neutral: \( u(x) = x \) for all citizens. Then the indirect utility function of citizen \( w \) at tax rate \( t \) is

\[
v(t; w) = (1-t)w + t \mu = w + t(\mu - w). \tag{3.1}
\]

Tax rates may be chosen in \([0, 1]\).

Now suppose that the distribution of voters, that is, of citizens who go to the polls on election day, is \( g_s(w) \), where \( s \) is a random variable (state) uniformly distributed on \([0, 1]\). Alternatively, we may interpret the model as saying that everyone votes, but that the distribution of preferences (i.e., of the parameter ‘\( w \’) is subject to a stochastic element.

Denote the mean of \( g_s \) by \( \mu_s \). Let \( G_s \) be the C.D.F. of \( g_s \). We shall suppose that \( G_s(\mu) \) is strictly decreasing in \( s \). Interpretation: ‘\( s \)’ is the weather, with larger ‘\( s \)’ meaning fouler weather. If the weather is foul, fewer poor people turn out to vote; thus \( G_s(\mu) \) is decreasing in \( s \).

Let \( t^1 > t^2 \) be two tax policies. It is obvious from (3.1) that the set of citizens who prefer \( t^1 \) to \( t^2 \), denoted \( W(t^1, t^2) \), is:

\[
W(t^1, t^2) = \{w < \mu \}. \tag{3.2}
\]

In state \( s \) the measure of this set is \( G_s(\mu) \). That is, \( G_s(\mu) \) is the fraction of voters who vote for \( t^1 \) over \( t^2 \) in state \( s \). Now \( t^1 \) defeats \( t^2 \) just in case this is a majority, i.e., when

\[
G_s(\mu) > \frac{1}{2}. \tag{3.3}
\]

By the italicized assumption of the previous paragraph, (3.3) is true just in case \( s < s^* \), where \( s^* \) is defined by:
Assuming that there is an \( s^* \in (0, 1) \) satisfying (3.4), then the probability that \( t^1 \) defeats \( t^2 \) is just \( s^* \), since \( s \) is uniformly distributed on \([0, 1]\).

Now suppose the L party represents a voter \( w_1 < \mu \). That voter’s expected utility, if party L proposes \( t^1 \) and party R proposes \( t^1 > t^2 \), is \( I^L(t^1, t^2) = s^*v(t^1; w_1) + (1-s^*)v(t^2; w_1) \), since with probability \( s^* \), \( t^1 \) wins, and with probability \( 1-s^* \), \( t^2 \) wins. Similarly, if the R party represents a voter \( w_R > \mu \), then its payoff function is \( I^R(t^1, t^2) = s^*v(t^1; w_R) + (1-s^*)v(t^2; w_R) \).

I next compute the Stackelberg equilibrium. Assume that L is the ‘incumbent’ and R is the ‘challenger’, where by definition, the challenger moves first. A Stackelberg equilibrium exists because the pay-off functions are continuous on the compact set \([0, 1]^2\). Let \( \tilde{t}^2 \) be R’s equilibrium policy, and assume \( \tilde{t}^2 < 1 \). Then L obviously maximizes \( I^L(t^1, \tilde{t}^2) \) at \( t^1 = 1 \). The same indeed holds if \( \tilde{t}^2 = 1 \).

Alternatively, suppose R is the incumbent. Let \( t^1 \) be any proposal; R maximizes \( I^R(t^1, t^2) \) by choosing \( t^2 = 0 \). Then L’s problem is to choose \( t^1 \) to maximize \( s^*v(t^1; w_1) + (1-s^*)v(t^2; w_1) \); the solution is \( t^1 = 1 \).

Hence, whether L is the incumbent or challenger, the equilibrium in the game of party competition involves the L party proposing a tax rate of unity. In sum:

**Proposition 3.1.** Let \( w_1 < \mu \), let \( G_\ast(\mu) \) be strictly decreasing in \( s \), and let \( u(x)=x \) be the universal von Neumann–Morgenstern utility function. Suppose there exists \( s^* \in (0, 1) \) such that \( G_\ast(\mu) = \frac{1}{2} \). Then, whether the party representing \( w_1 \) is the incumbent or challenger, the unique electoral equilibrium in the game of party competition entails \( t^1 = 1 \).

Proposition 3.1 sets the stage for our study. Will two-dimensional politics cause the Left party to compromise the radical redistributive policy it advocates when only income is the issue?

### 4. The two-dimensional politico-economic environment

We now suppose there are two issues, taxation and ‘religion.’ A citizen with religious view ‘\( a \)’ has a von Neumann–Morgenstern utility function \( u(x, z; a) = x - (\alpha/2)(z-a)^2 \), where \( x \) is after-tax wealth and \( z \) is the government’s religious policy. The positive number \( \alpha \) shall be called the salience of the religious issue. The joint distribution of wealth and religious views is represented by a density \( h(w, a) \) on \( \mathcal{A} \). The indirect utility function of voter \( (w, a) \) at policy \( (t, z) \), where \( t \) is a proportional tax rate, is

\[
v(t, z; w, a) = (1-t)w + t\mu - \frac{\alpha}{2} (z-a)^2.
\]
where $\mu$ is mean wealth. From Eq. (4.1), we may compute that voter $(w, a)$ prefers policy $\tau_L = (t_L, z_L)$ to $\tau_R = (t_R, z_R)$, iff:

$$
\begin{align*}
\tilde{z} + \frac{\Delta t(w - \mu)}{\alpha \Delta z} &> a & \text{if } \Delta z > 0, & (4.2a) \\
\tilde{z} + \frac{\Delta t(w - \mu)}{\alpha \Delta z} &< a & \text{if } \Delta z < 0, & (4.2b) \\
w < \mu & \text{ if } \Delta z = 0 \text{ and } \Delta t < 0, & (4.2c) \\
w > \mu & \text{ if } \Delta z = 0 \text{ and } \Delta t > 0, & (4.2d)
\end{align*}
$$

where $\Delta z = z_R - z_L$, $\Delta t = t_R - t_L$, and $\tilde{z} = (z_L + z_R)/2$.

I will assume that $h(w, a) = g(w)r(a, w)$, where $g(w)$ is a density on $W$ and, for each $w$, $r(a, w)$ is a density on $R$. The interpretation is that the wealth distribution of the population is given by $g$, and the distribution of religious views at wealth $w$ is given by $r(a, w)$. It shall be important that wealth and religious views are not independently distributed.

The stochastic element in elections is as in Section 3. A random variable, ‘$s$,’ which I shall assume is uniformly distributed on $[0, 1]$, determines the distribution of traits among those who show up at the polls. I shall assume that, in state $s$, the distribution of voters is given by:

$$
h_s(w, a) = g_s(w)r(a, w); \quad (4.3)
$$

the interpretation is that $s$ affects only the wealth distribution of the active electorate, but a representative sample of religious views shows up at each wealth level at the polls in every state of the world. Again, the interpretation may be that $s$ is a measure of the weather’s foulness.

The coalition of voters $W(\tau_L, \tau_R)$ who prefer $\tau_L$ to $\tau_R$ is given by (4.2). Thus the measure of voters who prefer $\tau_L$ to $\tau_R$ if, for instance, $\Delta z > 0$, is, from (4.2a):

$$
H_s(W(\tau_L, \tau_R)) = \int_{-\infty}^{\infty} g_s(w)r(a, w)\, da\, dw, \quad (4.4)
$$

where $H_s$ is the probability measure with density $h_s$.

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*The reader might inquire: Why not analyze an easier problem, in which parties have no uncertainty about the distribution of voter preferences? Aside from the fact that it is more reasonable to assume that parties are uncertain, it is easily observed that, in a two-dimensional political contest under certainty, equilibria usually fail to exist – Nash or Stackelberg. This is because there is usually no Condorcet winner, and the best response correspondence is almost always empty. Indeed, nothing in what follows rests on the choice in (4.3) to place all the uncertainty on the distribution of voter income. The assumption just simplifies some of the formulae to follow.*
Let \( \Phi(z, s) \) be the (cumulative) distribution function for religious views in state \( s \); that is,

\[
\Phi(z, s) = \int_{w=\infty}^{z} g_s(w) r(a, w) \, da \, dw.
\]

We assume:

\textbf{A1.} For any \( z \), \( \Phi(z, s) \) is strictly decreasing in \( s \).

\textbf{A1} plays the role that the assumption that \( G_s(\mu) \) was decreasing in \( s \) played in Section 3. If the rich tend to be more religious than the poor, and the fraction of rich voters increases with \( s \) (as when high \( s \) means foul weather on election day), then \textbf{A1} will surely hold.\(^5\)

Policy \( \tau_L \) defeats \( \tau_R \) in just those states \( s \) that \( H_s(W(\tau_L, \tau_R)) > \frac{1}{2} \). (We needn’t worry about what happens if \( H_s(W(\tau_L, \tau_R)) = \frac{1}{2} \), an event with zero probability.) It follows from \textbf{A1} and (4.4) that \( H_s(W(\tau_L, \tau_R)) > \frac{1}{2} \) just in case \( s < s^*(\tau_L, \tau_R) \), where \( s^*(\tau_L, \tau_R) \) is defined uniquely by:

\[
\int_{w=\infty}^{z+\frac{\Delta(t(e-w))}{\sigma \Delta z}} g_s(w) r(a, w) \, da \, dw = \frac{1}{2}.
\]

Thus, the probability that \( \tau_L \) defeats \( \tau_R \) is the probability of the event \( \{ s < s^* \} \), which is \( s^*(\tau_L, \tau_R) \), since \( s \) is uniformly distributed on \([0, 1]\).

That is, letting \( \pi(\tau_L, \tau_R) \) be the probability that \( \tau_L \) defeats \( \tau_R \) where \( \tau_R > \tau_L \), we have:

\[
\pi(\tau_L, \tau_R) = \begin{cases} 
1 & \text{if } H_s(W(\tau_L, \tau_R)) > \frac{1}{2} \\
 s^*(\tau_L, \tau_R) & \text{if } H_s(W(\tau_L, \tau_R)) = \frac{1}{2} \\
0 & \text{if } H_s(W(\tau_L, \tau_R)) < \frac{1}{2}.
\end{cases}
\]

More completely, we may write the function \( \pi(\tau_L, \tau_R) \) for all possible cases, using (4.2), as follows. Let \( \lambda \) be Lebesgue (uniform) measure on \([0, 1]\). Then:

\(^5\)Indeed, \textbf{A1} can be relaxed, but at the cost of computational complexity.
\[ \pi(\tau_L, \tau_R) = \begin{cases} 
\lambda \left( \int_w \int_a \frac{\Delta f(w, a)}{\eta(a) \Delta z} g(w) \rho(a, w) \, da \, dw > \frac{1}{2} \right) & \text{if } \Delta z > 0, \\
\lambda \left( \int_w \int_a \frac{\Delta f(w, a)}{\eta(a) \Delta z} g(w) \rho(a, w) \, da \, dw > \frac{1}{2} \right) & \text{if } \Delta z > 0, \\
1 \frac{\Delta f(w, a)}{\eta(a) \Delta z} g(w) \rho(a, w) \, da \, dw > \frac{1}{2} & \text{if } \Delta z = \Delta t = 0. 
\end{cases} \]

It may be verified that, since \( g_f(w) \) is continuous in \( s \) and \( w \) and \( \rho(a, w) \) is continuous, the function \( \pi \) is continuous except on the subset \( V = \{ \Delta z = 0, \Delta t \} \) of the domain \( T \times T \), where \( T = [0, 1] \times \mathbb{R} \) is the issue space.

Let the Left party represent a voter \((w_L, a_L)\) and the Right party a voter \((w_R, a_R)\), where \( w_L < \mu < w_R \) and \( a_L < a_R \). Recall that the parties’ pay-off functions are specified by (2.1). It is easily verified that the functions \( \Pi^R \) and \( \Pi^L \) are everywhere continuous on \( T \times T \); the discontinuity of \( \pi \) on the subspace \( V \) of the domain, defined above, turns out not to matter, since on \( V \), \( v(\tau_L; w, a) = v(\tau_L; w, a) \) for any \((w, a)\).

5. Analysis of Stackelberg equilibrium

Now think of the salience parameter \( \alpha \) in the utility function as variable, with \( \alpha \in [0, \infty) \). It follows from the continuity of the payoff functions that, for any \( \alpha \), there is a Stackelberg equilibrium for the game \( Q = (a, (a_L, w_L), (a_R, w_R), g, r, \{g_i\}, \nu) \). We assume Left is the follower.

We next assume:

Although the strategy space for each player, \([0, 1] \times \mathbb{R}\), is not compact, one can show that the payoff functions \( \Pi^R(\cdot, \tau_R) \) and \( \Pi^L(\tau_L, \cdot) \), are decreasing outside a compact set, and existence follows.
A2. (a) In the game $\mathcal{G}_a$ (i.e., when $u(x, z, w, a) = -(z - a)^2$), there is a finite number of Stackelberg equilibria. For any such equilibrium $(z^*_L, z^*_R)$, we have $a_L < z^*_L < z^*_R$, and $0 < \pi(z^*_L, z^*_R) < 1$.

(b) For any equilibrium policy $z^*_R$ in $\mathcal{G}_a$, L’s best response is unique.

A2 is simply a non-degeneracy axiom about the one-dimensional game $\mathcal{G}_a$. For the analysis of one-dimensional games, which justifies this claim, see Roemer (1997a).

Denote the payoff to party L in the game $\mathcal{G}_a$ at the policy pair $(t_1, t_2)$ as $P(t_1, t_2)$ with the analogous notation for party R. Let $\Theta(\alpha)$ be the Stackelberg equilibrium correspondence, which associates to any $\alpha$ the Stackelberg equilibria of the game $\mathcal{G}_a$. We have the following two facts:

**Proposition 5.1.** Let A2(b) hold. Then $\Theta(\alpha)$ is upper-hemi-continuous at $\alpha = \infty$.

**Proof:** See Appendix A.

Let $(\tau_L(\alpha), \tau_R(\alpha))$ be a continuum of equilibria for the games $\mathcal{G}_a$, $\alpha < \infty$, where $\tau_L(\alpha) = (t_1(\alpha), z_1(\alpha))$.

**Proposition 5.2.** Let A2(a) hold. For sufficiently large $\alpha$:

(a) $\Delta z(\alpha) > 0$ and $\Delta z(\alpha)$ is bounded away from 0;

(b) $\bar{z}(\alpha) - a_L$ is positive and bounded away from zero.

**Proof:** See Appendix A.

Our task is to find a condition under which, for sufficiently large $\alpha$, at the Stackelberg equilibria of $\mathcal{G}_a$, $t_1(\alpha) = 0$: that is, the Left will propose tax rates of zero! We next state that condition, and then our theorem.

Let $(z_L(\infty), z_R(\infty))$ be any equilibrium in the game $\mathcal{G}_a$, and $\Delta z(\infty) = z_R(\infty) - z_L(\infty)$. Let $s^*$ be the probability of victory of party L at this equilibrium. Define the number $\bar{\mu} = \bar{\sigma} - \hat{\sigma}$, where $\bar{\sigma} = \int w g_v(z) r(\bar{z}(\infty), w) dw$, and $\hat{\sigma} = \int w g_v(z) r(\bar{z}(\infty), w) dw$. By definition, $\bar{\mu}$ is the mean wealth of the cohort of voters with religious position $\bar{z}(\infty)$ in the state $s^*$. Our condition is:

**A3.** For all Stackelberg equilibria in the game $\mathcal{G}_a$, we have:

$$\bar{\mu} - \mu > -\frac{(\mu - w_L)\Delta z(\infty)}{2(z_L(\infty) - a_L)}$$

**Theorem 5.1.** Suppose A1, A2, and A3 hold. Then for all sufficiently large $\alpha$, all Stackelberg equilibria of the game $\mathcal{G}_a$ have $t_1(\alpha) = 0$. 

**Proof:** See Appendix A.

**Definition 5.1.** Let \( a^m(s) \) be the median religious view in state \( s \). For any \( \delta > 0 \), we say **uncertainty is less than** \( \delta \) iff there is a number \( \nu \) such that, for all \( s \), \( a^m(s) \) lies in a \( \delta \) interval around \( \nu \).

If uncertainty is sufficiently small, in the above sense, then \( \Delta(z(\infty)) = 0 \): in the one-dimensional game \( \mathcal{G}_w \), both \( z_1(\infty) \) and \( z_2(\infty) \) will be very close to the median religious view in state \( s^* \), as will be their average \( \bar{z} \). Thus, \( \bar{\mu} \) is approximately equal to the median wealth of the cohort of voters who have the median religious view in state \( s^* \). But since \( \Delta(z(\infty)) = 0 \), (5.7) is true as long as \( \bar{\mu} > \mu \). Thus a sufficient condition for the truth of (5.7) is that uncertainty be small and

\[
\text{the mean wealth of the cohort of voters with the median religious view in all states is greater than mean wealth of the population.} \quad (*)
\]

Thus, we have:

**Corollary.** If \( A1 \) and \( A2 \) hold, uncertainty is small, the mean wealth of the cohort of voters with the median religious view in all states is greater than mean wealth of the population, and the religious issue is sufficiently salient, then Labour will propose a zero tax rate in all Stackelberg equilibria.

Although the analysis leading to this corollary is not the simplest, condition \( (*) \) is a simple one, which can be empirically tested, as I attempt in Section 8 below. We indeed need to know very little about the distribution of preferences to check whether \( (*) \) holds. The fact that, in the final analysis, we do not need to know much about the joint distribution of \( (w, a) \) to decide whether increasing salience of the religious issue will lead to increasing economic conservatism of the Left party has been purchased by, *inter alia*, assuming a simple form for the utility function – that it be quasi-linear in income, and Euclidean in the religious dimension. Introducing a more complex utility function appears not to lead to a simple condition like \( (*) \).

### 6. A new equilibrium concept based on internal party struggle

As I explained in Section 1, pure-strategy Nash equilibria do not generally exist in the two-dimensional game between parties with pay-off functions specified in (2.1). I here introduce a new specification of party preferences, under which pure-strategy Nash equilibria will exist.

The idea is based upon European party history. We will assume that there are three factions in each party: reformists, opportunists, and militants. Reformists have the preferences given by \( \Pi^r \) and \( \Pi^p \): they wish to maximize the expected utility of the party’s constituents. Opportunists have preferences given by \( \pi \) (for
Left), and $1 - \pi$ (for Right): they wish only to maximize the probability of victory. Opportunists are the characters who dominate Anthony Downs’s (Downs, 1957) view of political competition. Finally, militants are not concerned at all with winning the election: Left’s militants which to maximize $v_L$ and the Right’s militants wish to maximize $v_R$. Thus, the militants are interested in advertising the preferences of their constituent; they view elections as a pulpit for announcing and propagating the party’s line. In this section, I shall assume that each party contains all three factions, and that each faction has the power to veto any proposal for the party’s platform. The equilibrium concept that will follow from this assumption I call party unanimity Nash equilibrium (PUNE).

I shall not here attempt to justify the historical basis of this approach, which I do in Roemer (1997b).

**Definition 6.1.** We say that Left agrees to deviate from $\tau_L \in T$ to $\tau'_L \in T$ at $\tau_R$ iff all factions in Left weakly prefer $(\tau'_L, \tau_R)$ to $(\tau_L, \tau_R)$ and at least one faction strictly prefers the former to the latter.

**Definition 6.2.** $(\tau_L, \tau_R)$ is a party unanimity Nash equilibrium iff there is no platform at $\tau_R$ to which Left agrees to deviate and there is no platform at $\tau_L$ to which Right agrees to deviate.

Formally, we can define a party’s (incomplete) preferences as the intersection of the preference relations of its three factions; that is, for Left,

$$(\tau'_L, \tau_R) \succeq_L (\tau_L, \tau_R)$$

if and only if

$$\pi(\tau'_L, \tau_R) \geq \pi(\tau_L, \tau_R)$$

and

$$P(\tau'_L, \tau_R) \geq P(\tau_L, \tau_R)$$

with a similar definition for $\succeq_R$. Then we can say that $(\tau_L, \tau_R)$ is a PUNE iff it is a Nash equilibrium with respect to the incomplete preferences $\succeq_L$ and $\succeq_R$. Denote the game with these preferences (of the parties) $G^*_a$ to be distinguished from the (Stackelberg) game $G_a$ studied in the last section, where the players have different preferences.

In this section, I shall show that if condition (*) holds, then for large $\alpha$, $t_L(\alpha) = 0$ in all non-trivial PUNE of $G^*_a$, where a non-trivial equilibrium is one in which neither party wins with probability one. Thus, our central result shall be robust with respect to a change in the equilibrium concept from Stackelberg to party-unanimity-Nash. Finally, I will show that non-trivial PUNE exist in these games.

Before beginning, it is useful to observe that, indeed, we may ignore the reformists in the definition of PUNE. That is:

**Lemma 6.1.** A pair of platforms constitutes a PUNE iff the militants and opportunists, in both parties, do not agree to deviate.
Proof: Follows quickly from the definitions. (Or see Roemer (1997b).)

To be precise, we must restate the critical axiom for the new equilibrium concept:

**A3**  For all non-trivial PUNE in the game $\mathcal{G}^*_\alpha$, we have:

$$\bar{\mu} - \mu > \frac{(\mu - w_L)\Delta z(\infty)}{2(z_L(\infty) - a_L)}.$$  \hspace{1cm} (6.7)

Define $\Theta^*(\alpha)$ as the PUNE correspondence; that is, $\Theta^*(\alpha)$ is the set of all PUNE for the game $\mathcal{G}^*_\alpha$.

Our line of argument is as follows.

**Lemma 6.2.** $\Theta^*$ is upper-hemi-continuous at $\alpha = \infty$.

Let $\{(t_L(\alpha), z_L(\alpha)), (t_R(\alpha), z_R(\alpha))\}$ be a sequence of non-trivial PUNE for $\alpha$ tending to infinity, which, by Lemma 6.2, converge to a PUNE, $(z_L(\infty), z_R(\infty))$ of $\mathcal{G}^*_\infty$. (We can ignore the tax rates in $\mathcal{G}^*_\infty$)

**Proposition 6.1.** If $A3^*$ holds, then for all sufficiently large $\alpha$, $t_L(\alpha) = 0$.

**Proposition 6.2.** Let $\varepsilon > 0$. If uncertainty is sufficiently small, then $z_R(\infty) - z_L(\infty) = \Delta z(\infty) < \varepsilon$.

Now suppose that uncertainty is small and condition (*) holds. Then it follows from Prop. 6.2 that (6.7) holds, and hence it follows from Proposition 6.1 that $t_L(\alpha) = 0$ for large $\alpha$. This proves:

**Theorem 6.1.** If uncertainty is small and condition (*) holds, and if $\alpha$ is sufficiently large, then in any non-trivial PUNE, $t_L(\alpha) = 0$.

I shall not provide the proof of Lemma 6.2, but the proofs of Propositions 6.1 and 6.2 are found in the appendix.

Finally, we prove that Theorem 6.1 is not vacuous:

**Theorem 6.2.** Let uncertainty be small and condition (*) hold. Then, for all large $\alpha$, non-trivial PUNE exist in the game $\mathcal{G}^*_\alpha$.

Proof: See Appendix A.

Before concluding this section, a further remark on the notion of PUNE is in order. One might object to the assumption that each faction in the party has veto power over the policy. Suppose, then, that each party works out a method of inner-party bargaining, or compromise. The point is that, whatever policies the two parties reach as a consequence of inner-party bargaining, that policy allocation is a PUNE. For it surely must be the case that, at that allocation, no parties’ factions would agree to deviate to another policy. Therefore, any such (bargaining)
equilibrium inherits all the properties of a PUNE — in particular, if condition (*) holds, then for large \( \alpha \), Left proposes a tax rate of zero. What is called into question, when we switch to a ‘bargaining’ concept of inner-party struggle, is existence of equilibrium.

7. Further discussion

I have shown that, if there is a non-economic issue which is sufficiently important to voters, if parties represent constituents who have preferences over taxation and the non-economic issue, and if assumption (*) holds and uncertainty is small, then in two kinds of electoral equilibrium, the tax policy of the Left party will be significantly less than unity. (Section 3 showed that when \( \alpha = 0 \), the Left always proposes a tax rate of one; so as \( \alpha \) increases, the tax rate eventually decreases towards zero.) The result is striking because it may simultaneously be true that the ideal tax rate for the majority of the population, in all states, is unity! This ‘paradox’ is due to the structure of political competition, which is party competition, in which the different dimensions of policy cannot be unbundled. While the ideal tax rate for the majority of a population may be unity, that tax rate will not be observed in equilibrium, even when one party represents (a sub-constituency of) that expropriation-desiring majority.

I will try to give some intuition for how condition (*) drives our result. If \( \alpha \) is large, then the game is essentially a one-dimensional game over religious policy. If uncertainty is small, then the median religious view varies little across states. In an equilibrium where both parties win with positive probability, both parties must therefore play a religious policy close to that approximately constant median religious view. We may even say that the cohort of the population who hold approximately the median religious view are the decisive voters. But if that cohort’s wealth is greater than mean population wealth, as condition (*) states, then their ideal tax rate is zero. Competition forces Left (and Right) to propose a tax rate of zero, to attract the decisive cohort. If you object to some slippage in this ‘argument,’ then read the proofs.

We may apply exactly the same analysis to determine when Right parties (who represent rich, religious voters) will, in Stackelberg or PUNE equilibrium, propose high tax rates. (In the Stackelberg case, we must assume that Right is the follower.) The key condition now turns out to be:

\[
\bar{\mu} - \mu < \frac{(w_R - \mu)\Delta z(\infty)}{2(\bar{z}_R(\infty) - a_R)}. \tag{7.1}
\]

Note that the r.h.s. of (7.1) is negative, so (7.1) will be satisfied if:
uncertainty is small, and for all states, the mean wealth of the cohort of voters with the median religious view is less than mean population wealth.

Under these conditions, when $\alpha$ is sufficiently large, Right will propose a tax rate of unity in either kind of equilibrium.

8. Empirical tests

For the United States, I suggest that ‘race’ is the prominent non-economic issue. Using the National Election Surveys, we computed whether the average income of voters who hold the median view on the race issue is greater than mean population income – to see whether condition (*) holds. Among the many questions asked in these Surveys is a ‘thermometer’ question on ‘Blacks.’ Respondents are asked to choose a number between 0 and 100 telling how ‘warmly’ or ‘favorably’ they feel about the issue. 100 is the warmest possible. In the question we used, the issue was simply stated as ‘Blacks.’ The results, for 1974–1994, are presented in Table 1.

Not all respondents in the NES are voters; in particular, the respondent is asked if he voted. We took the mean population income ($\mu$) to be the mean reported income of all respondents in the survey (col. 1 of Table 1). Col. 2 of the table gives the mean income of voters (which we do not use in our statistical test). Col. 4 gives the median thermometer value of all voter responses on the Black issue.

---

Table 1

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1974</td>
<td>$12,730</td>
<td>$14,296</td>
<td>$15,043</td>
<td>65.07</td>
<td>$9745</td>
<td>$10,104</td>
<td>$10,572</td>
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<tr>
<td>1976</td>
<td>$14,628</td>
<td>$15,929</td>
<td>$17,964</td>
<td>61.08</td>
<td>$10,719</td>
<td>$11,051</td>
<td>$11,774</td>
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<tr>
<td>1980</td>
<td>$20,955</td>
<td>$22,729</td>
<td>$23,357</td>
<td>64.46</td>
<td>$15,041</td>
<td>$15,236</td>
<td>$15,792</td>
</tr>
<tr>
<td>1982</td>
<td>$22,734</td>
<td>$24,482</td>
<td>$25,054</td>
<td>63.7</td>
<td>$15,959</td>
<td>$15,905</td>
<td>$12,937</td>
</tr>
<tr>
<td>1984</td>
<td>$25,402</td>
<td>$27,911</td>
<td>$29,458</td>
<td>65.01</td>
<td>$18,806</td>
<td>$19,375</td>
<td>$19,715</td>
</tr>
<tr>
<td>1986</td>
<td>$28,412</td>
<td>$31,896</td>
<td>$33,089</td>
<td>67.37</td>
<td>$20,439</td>
<td>$21,143</td>
<td>$20,860</td>
</tr>
<tr>
<td>1990</td>
<td>$31,262</td>
<td>$35,977</td>
<td>$38,233</td>
<td>71.31</td>
<td>$23,980</td>
<td>$24,575</td>
<td>$24,810</td>
</tr>
<tr>
<td>1992</td>
<td>$35,751</td>
<td>$39,567</td>
<td>$40,277</td>
<td>65.57</td>
<td>$26,836</td>
<td>$27,209</td>
<td>$26,479</td>
</tr>
<tr>
<td>1994</td>
<td>$37,727</td>
<td>$43,263</td>
<td>$46,087</td>
<td>64.33</td>
<td>$27,864</td>
<td>$28,713</td>
<td>$31,733</td>
</tr>
</tbody>
</table>

(*) Range [0, 100], where the higher the number the more favorable the agent feels toward blacks’ issues.

I thank research assistants Woojin Lee and Humberto Gonzalez–Llavador for carrying out the data analysis in this section.
and col. 3 gives the mean income of this median cohort (μ). It is evident that μ > μ; in fact, using a central-limit-theorem test, we computed that for the last four election years (1988 on), μ > μ at the 0.999 significance level.

It is not surprising (compare columns 1 and 2 of Table 1) that the mean income of voters is greater than population mean income. But it is not this fact alone which explains our result, since we note that, in every year, the mean income of the median cohort of voters is greater than the mean income of voters, as well.

Regarding the salience of non-economic issues for the American electorate, George Gallup (of the Gallup Poll) says: “[Americans] are more concerned about the state of morality and ethics in their nation than at any time in the six decades of scientific polling.”

We attempted to test whether the salience of non-economic issues has been increasing, as follows. The National Election Survey asks each respondent to list the three most important issues, in his view. There are hundreds of acceptable answers to this question, coded in the NES. We coded these issues as ‘economic issues,’ ‘values issues,’ or ‘other issues,’ and defined the salience of values, for a cohort, as the number of values issued mentioned divided by the number of economic issues mentioned in the answer to this question. Table 2 gives the salience rate, so computed, for the election years 1974–1994.

Evidently, Gallup’s view is borne out: never, in the past twenty years, has the salience of values issues been higher than in 1994. In fact, the salience of values issues has been rising steadily during this period, in the United States, except for a decline in the period 1988–1992, roughly corresponding to a recession.

The British Social Attitudes Survey is the annual counterpart, in the UK, of the US General Social Survey. In 1993, the BSAS asked a series of questions designed to ascertain the respondents’ views on the authoritarian–libertarian dimension. The respondent was asked to mark his degree of agreement, on a scale of one to five

Table 2
Salience of values issue, US electorate

<table>
<thead>
<tr>
<th>Year</th>
<th>Salience Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1974</td>
<td>0.312</td>
</tr>
<tr>
<td>1976</td>
<td>0.331</td>
</tr>
<tr>
<td>1980</td>
<td>0.397</td>
</tr>
<tr>
<td>1982</td>
<td>0.539</td>
</tr>
<tr>
<td>1984</td>
<td>0.605</td>
</tr>
<tr>
<td>1986</td>
<td>0.921</td>
</tr>
<tr>
<td>1988</td>
<td>1.236</td>
</tr>
<tr>
<td>1990</td>
<td>0.929</td>
</tr>
<tr>
<td>1992</td>
<td>0.595</td>
</tr>
<tr>
<td>1994</td>
<td>2.109</td>
</tr>
</tbody>
</table>

9The Economist, November 11–17, 1995, p. 29.
(strongly agree, agree, neither agree nor disagree, disagree, strongly disagree) with the following statements:

a) It is right that young people should question traditional British values;

b) British courts generally give sentences that are too harsh;

c) The death penalty is never an appropriate penalty;

d) Schools should teach children to question authority;

e) There are times when people should follow their conscience, even if it means breaking the law.

It is important to note that we do not have information on voters in the British data, only on the general population.

We coded the answers one to five, and assigned each respondent an average value, including in the sample only respondents who answered at least three of the five questions. We then computed the median cohort, whose response was 2.67, lying between ‘agree’ and ‘neither agree nor disagree’. We computed the mean income of the median cohort, and the mean income of the sample.

In Table 3, I report the statistical features of the answers to these questions that are relevant for us. This time, the mean income of the median cohort appears to be less than mean income of the sample; the central-limit-theorem test says that this order of the two means is correct with probability 0.78 – not a very high confidence level. One must note, however, that we do not have the mean income of the median voter cohort, which may be greater than mean population income. If, however, we assume that \( \mu - \mu < 0 \) is true, then, from the discussion of Section 7, the relevant hypothesis is not about the behavior of the Labour Party but rather the Conservative Party. The inference is that, with probability 0.78, inequality (7.1) holds, and the model, in that case, implies that a Conservative Party in power would move to the left in its economic policy as the salience of the authoritarian-libertarian issue increases.

From these tests, the model suggests that, if the salience of the non-economic issue of race increases in the United States, Democrats would propose increasingly conservative tax policies, while we have no reason to believe that Republicans would propose increasingly liberal tax policy. We have somewhat weaker reason to believe that, as the salience of the authoritarian–libertarian issue increases in Britain, the Conservative Party would move to the left in its economic policies.

<table>
<thead>
<tr>
<th>Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>British Social Attitudes Survey, 1993 Authoritarian vs. Libertarian preferences</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample size</th>
<th>2100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean income of sample (μ)</td>
<td>15 194</td>
</tr>
<tr>
<td>Median view on issue</td>
<td>2.667</td>
</tr>
<tr>
<td>Mean income of median cohort (m)</td>
<td>14 691</td>
</tr>
<tr>
<td>Size of median cohort (n)</td>
<td>219</td>
</tr>
<tr>
<td>S.D. of median cohort’s income (β)</td>
<td>9777</td>
</tr>
<tr>
<td>Pr(m &lt; μ)</td>
<td>0.78</td>
</tr>
</tbody>
</table>
9. Concluding remark

We may finally reflect upon a view, which has often been held in Left circles, that the Right deliberately ‘creates’ a certain non-economic issue – or tries to increase the salience of some such issue for voters – as a means of pulling working-class voters away from Left parties, thereby driving economic policies to the right. In this view, the Right party pretends to care about the ‘religious’ issue, while in fact being interested only in lowering tax rates (or rolling back nationalization, etc.). Right may implement this masquerade by attracting political candidates who do, indeed, feel strongly on the ‘religious’ issue.

Our analysis certainly indicates that this can be a strategy to achieve more conservative economic policy. Of course, Left can play the same game, and attempt to increase the salience of an issue for which ‘\( \bar{\mu} < \mu \)’ holds, thus forcing Right to move to the left on economic policy. Our analysis, then, suggests a new way to read the history of the development of non-economic issues in electoral politics. Have Left and Right parties ‘chosen’ which non-economic issues to emphasize (i.e., increase the salience of) with an eye towards pushing electoral equilibrium on the economic dimension in a desired direction?

Whatever the verdict on that historical issue, our analysis suggests that emerging new dimensions of citizen concern, which are addressed in competitive, party politics, can change the positions of parties on classical issues in surprising ways.

Acknowledgements

The idea for this paper was, I think, hatched in a discussion with Ignacio Ortuno-Ortin, during my visit to the University of Alicante in 1994. I am also grateful to him for finding errors in an earlier version. I wish, as well, to thank anonymous referees for their advice, and Woojin Lee and Humberto Gonzalez-Llavador for expert research assistance.

Appendix A

Proof of Proposition 5.1: Let \((\tau_l(\alpha), \tau_R(\alpha))\) be a sequence of Stackelberg equilibria in the games \( \mathcal{G}_\alpha \), and let \( z_l(\alpha) \) and \( z_R(\alpha) \) converge to \( z_L(\infty) \) and \( z_R(\infty) \), respectively. Suppose, contrary to the claim, that \((z_l(\infty), z_R(\infty))\) is not a Stackelberg equilibrium in \( \mathcal{G}_L \). A standard continuity argument establishes that

\[ \text{in the game } \mathcal{G}_\alpha \text{ the tax policies are irrelevant, so we do not refer to them in describing equilibria of } \mathcal{G}_L. \]
\[ z_L(\infty) \text{ must be a best response to } z_R(\infty); \text{ so it must therefore be that there exists an equilibrium pair } (\tilde{z}_L, \tilde{z}_R) \text{ such that } \tilde{z}_L \text{ is a best response to } \tilde{z}_R \text{ and } \]
\[ II^R(\tilde{z}_L, \tilde{z}_R; \infty) > II^R(\tilde{z}_L(\infty), \tilde{z}_R(\infty); \infty). \]

Let \((\hat{t}_L(\alpha), \hat{z}_L(\alpha))\) be L's best response to \((t_R(\alpha), \tilde{z}_R)\) in \(\mathcal{G}_a\). Then \(\hat{z}_L(\infty) = \lim_{\alpha \to \infty} \hat{z}_L(\alpha)\) is a best response to \(\tilde{z}_R\) in \(\mathcal{G}_a\). By A2(b), \(\hat{z}_L(\infty) = \tilde{z}_L\). Hence \(II^R((\hat{t}_L(\alpha), \hat{z}(\alpha)), (t_R(\alpha), \tilde{z}_R)); \alpha)\) approaches \(II^R(\tilde{z}_L, \tilde{z}_R; \infty)\) as \(\alpha\) approaches \(\infty\). In particular, by the above inequality, for large \(\alpha\):
\[ II^R((\hat{t}_L(\alpha), \hat{z}(\alpha)), (t_R(\alpha), \tilde{z}_R); \alpha) > II^R((t_L(\alpha), \tilde{z}_L(\alpha)), (t_R(\alpha), \tilde{z}_R(\alpha)); \alpha). \]

This contradicts the fact that \(((t_L(\alpha), z_L(\alpha)), (t_R(\alpha), \tilde{z}_R(\alpha)))\) is a Stackelberg equilibrium in \(\mathcal{F}_a\), which establishes the claim. ■

**Proof of Proposition 5.2:** By the upper-hemi-continuity of the equilibrium correspondence \(\Theta(\alpha)\) at \(\infty\), any converging subsequence of the continuum \((\tau_1(\alpha), \tau_R(\alpha))\) converges to an equilibrium of \(\mathcal{G}_a\). The claims follow immediately from A2(a). ■

**Proof of Theorem 5.1:** Suppose to the contrary: that for a sequence of \(\alpha\)'s tending to infinity, there is a Stackelberg equilibrium of \(\mathcal{F}_a\), in which \(t_L(\alpha) > 0\). We know that \(\Delta(\alpha) > 0\) by Proposition 5.2; hence, for large \(\alpha\), \(\pi(\tau_L(\alpha), \tau_R(\alpha))\) is indeed given by (4.6), and hence, either \(\pi(\tau_L(\alpha), \tau_R(\alpha))) = s^*(\tau_L(\alpha), \tau_R(\alpha))\), where \(s^*\) is defined by (4.5), or \(\pi(\tau_L(\alpha), \tau_R(\alpha))) \in [0, 1]\). But by A2(a), since for all equilibria of the game \(\mathcal{G}_a\), \(\pi \in [0, 1]\), it follows that for sufficiently large \(\alpha\), \(\pi(\tau_L(\alpha), \tau_R(\alpha))) \in [0, 1]\), and therefore \(\pi(\tau_L(\alpha), \tau_R(\alpha))) = s^*(\tau_L(\alpha), \tau_R(\alpha))\).

Differentiating (4.5) implicitly w.r.t. \(t_L\), we may write:
\[ \frac{\partial s^*}{\partial t_L} = \int w \int_{-\infty}^{\infty} \frac{\partial g_{*w}(w) r(\bar{z}, \Delta, t, \alpha \Delta z)}{\partial s} (w - \mu) dw \]
\[ \int_{-\infty}^{\infty} \frac{\partial s^*}{\partial s} (w) r(\alpha, w) da dw, \]

as long as the denominator in (Ap.1) does not vanish, where I have omitted the argument \(\alpha\) on the variables \(\bar{z}, \Delta, t, \alpha \Delta z\). But Axiom A1 tells us that the expression \(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial g_{*w}(w)}{\partial s} (w - \mu) dw \) is strictly less than zero, since this expression is just the derivative of \(\Phi(\bar{z}, s)\) w.r.t. \(s\), and so the denominator of (Ap.1) does not vanish.

We assume that L is the incumbent and R is the challenger (i.e., R moves first). Since \(s^*\) is differentiable for large \(\alpha\), so is \(II^L(\tau_L(\alpha), \tau_R(\alpha); \alpha)\) differentiable at \((\tau_L(\alpha), \tau_R(\alpha))\) for large \(\alpha\). Since \(\tau_L(\alpha)\) is a best response to \(\tau_R(\alpha)\), it therefore follows that \(\frac{\partial II^L}{\partial \tilde{z}_L} (\tau_L(\alpha), \tau_R(\alpha), \alpha) = 0\), since \(z_L(\alpha)\) is an interior solution (as
the domain of possible \( z_i \)'s is the real line). This first-order condition can be solved to yield:

\[
\frac{\partial s^*}{\partial z_L} = \frac{a \delta^* (z_L - a_L)}{\Delta (w_L - \mu) + \alpha \Delta z (\bar{z} - a_L)}. 
\] (Ap.2)

Similarly, it follows that \((\partial \Pi^L / \partial t_L) \geq 0\), since by hypothesis \( t_L(\alpha) > 0 \) for all (finite) \( \alpha \). The just stated inequality can be solved to yield:

\[
\frac{\partial s^*}{\partial t_L} = \frac{s^* (w_L - \mu)}{\Delta (w_L - \mu) + \alpha \Delta z (\bar{z} - a_L)}. 
\] (Ap.3)

an expression whose derivation uses the fact that the denominator of (Ap.3) is positive, which follows from Proposition 5.2.\( ^{11} \)

Next, differentiating (4.5) w.r.t. \( z_L \) yields:

\[
\frac{\partial s^*}{\partial z_L} = -\int_w g_{s^*}(w) r(\bar{z} + \frac{\Delta t}{\alpha \Delta z} (w - \mu), w) \left( \frac{1}{2} + \frac{\Delta (w - \mu)}{\alpha (\Delta z)^2} \right) dw 
\]

\[
\int_{\bar{z} + \frac{\Delta t}{\alpha \Delta z} (w - \mu)}^{\bar{z}} \int_{-\infty}^{w} \frac{\partial g_{s^*}(w)}{\partial s} r(a, w) da dw 
\]

Let the (common) denominator in the fractions on the r.h.s. of (Ap.4) and (Ap.1) be denoted ‘D.’ Using (Ap.4) and (Ap.2), we can solve for D, eliminating \((\partial s^* / \partial z_L)\); substituting the expression for D into (Ap.1) yields:

\[
\frac{\partial s^*}{\partial t_L} = \int_w g_{s^*}(w) r(\bar{z} + \frac{\Delta t}{\alpha \Delta z} (w - \mu), w) \left( \frac{1}{2} + \frac{\Delta (w - \mu)}{\alpha (\Delta z)^2} \right) dw 
\]

\[
\int_{\bar{z} + \frac{\Delta t}{\alpha \Delta z} (w - \mu)}^{\bar{z}} \int_{-\infty}^{w} \frac{\partial g_{s^*}(w)}{\partial s} r(a, w) da dw 
\]

In turn, (Ap.5) and (Ap.3) imply:

\[
\int_w g_{s^*}(w) r(\bar{z} + \frac{\Delta t}{\alpha \Delta z} (w - \mu), w) \left( \frac{w - \mu}{\alpha \Delta z} \alpha (z_L - a_L) \right) dw 
\]

\[
\int_w g_{s^*}(w) r(\bar{z} + \frac{\Delta t}{\alpha \Delta z} (w - \mu), w) \left( -\frac{1}{2} - \frac{\Delta (w - \mu)}{\alpha (\Delta z)^2} \right) dw 
\]

\[\theta \]

Establishing the positivity of the denominator of (Ap.3) also uses the fact that \( \Delta t \leq 0 \) at equilibrium, which is not proved here, though it is true.
or
\[
(z - a_1) \alpha \Delta z \int \frac{g_s(w)r(z + \frac{\Delta t}{\alpha \Delta z} (w - \mu), w)(w - \mu)}{w} \, dw
\]

\[
\int g_s(w)r(z + \frac{\Delta t}{\alpha \Delta z} (w - \mu), w) \left( \frac{-\alpha (\Delta z)^2}{2} - \Delta t (w - \mu) \right) \, dw
\]

\[
\geq - (\mu - w_1).
\]  
(Ap.6)

Letting \( \alpha \to \infty \), (Ap.6) becomes, in the limit:

\[
2(z - a_1) \int g_s(w)r(z(\infty), w)(w - \mu) \, dw
\]

\[
\Delta z(\infty) \int g_s(w)r(z(\infty), w) \, dw
\]

where we use the fact that \( \Delta z(\alpha) \) is bounded away from zero (Proposition 5.2) so \( \alpha \Delta z(\alpha) \to \infty \).

Using the definitions of \( \bar{\rho}, \bar{\sigma} \) and \( \bar{\mu} \) provided in the text, we can write the negation of (Ap.7) as

\[
\bar{\mu} - \mu > \frac{(\mu - w_1) \Delta z(\infty)}{2(z - a_1)}
\]

which is precisely condition (5.7). Hence, by A3, (Ap.7) does not hold, which contradicts the original supposition – that there is a sequence of equilibria at which \( t_1(\alpha) > 0 \), and the theorem is proved.

**Proof of Proposition 6.1:** Suppose there is a sequence of \( \alpha \)'s tending to infinity, with \( t_1(\alpha) > 0 \). We shall show that, at each sufficiently large \( \alpha \), there is a direction in which Left’s militants and opportunists will agree to deviate, which, by Lemma 6.1, contradicts the assumption that we are at a PUNE.

To be specific, we shall show the existence, for large \( \alpha \), of a direction \((-1, \delta(\alpha))\) such that:

\[
\nabla v_L \cdot (-1, \delta(\alpha)) > 0.
\]  
(Ap.9)

and

\[
\nabla L s^* \cdot (-1, \delta(\alpha)) > 0.
\]  
(Ap.10)

which means that both the militants and opportunists in Left can increase their utility by moving in the direction \((-1, \delta(\alpha))\). Recall that the components of the gradient \( \nabla L s^* = (\partial s^*/\partial t_1, \partial s^*/\partial z_L) \) are given by equations (Ap.1) and (Ap.4).
Since $t_1(\alpha)>0$, the direction $(-1, \delta(\alpha))$ is feasible at $\tau_1$, for any number $\delta(\alpha)$. (Ap.9) expands to:

$$w_L - \mu - \delta(\alpha)\alpha(z_L(\alpha) - a_L) > 0,$$

which we rewrite as:

$$\delta(\alpha) < \frac{w_L - \mu}{\alpha(z_L(\alpha) - a_L)^2}.$$  \hspace{1cm} \text{(Ap.9')}

For the moment, let us choose $\delta(\alpha) = (w_L - \mu)/\alpha(z_L(\alpha) - a_L))$. Substituting this value into the inequality (Ap.10), using the formulae for the components of $\nabla_s s^*$, and taking the limit of the derived expression as $\alpha$ goes to infinity, we may compute that (Ap.10) holds for large $\alpha$ if:

$$\int g_{s^*}(w)r(\tilde{z}, w) \frac{w - \mu}{\Delta(z)} \, dw + \frac{w_L - \mu}{2(z_L(\alpha) - a_L)} \int g_{s^*}(w)r(\tilde{z}, w) \, dw > 0.$$

\hspace{1cm} \text{(Ap.11)}

But (Ap.11) is equivalent to inequality (5.7); hence $A^3*$ implies the truth of (Ap.10), for this choice of $\delta(\alpha).$ $^{12}$

It follows that if we choose $\delta(\alpha) = (w_L - \mu)/\alpha(z_L(\alpha) - a_L)) - \varepsilon$, for $\varepsilon$ sufficiently small, then both (Ap.9) and (Ap.10) hold, which is the desired contradiction. $\blacksquare$

\textbf{Proof of Proposition 6.2:} The game $G^*_\alpha$ is played on a one-dimensional strategy (issue) space, and the non-trivial PUNE for this game are easy to characterize. Consider the interval defined by the values $a^*(s)$, for $s \in [0, 1]$; if uncertainty is sufficiently small, then this interval becomes arbitrarily small. If $(z_L^*, z_R^*)$ is a non-trivial PUNE, then $z_L^*$ and $z_R^*$ must both lie in the interior of this interval, which proves the claim.

\textbf{Proof of Theorem 6.2:} Let $(z_L^*, z_R^*)$ be non-trivial PUNE in the game $G^*_\alpha$. I shall argue that $((0, z_L^*), (0, z_R^*))$ is not a PUNE in $G^*_\alpha$, for large $\alpha$. It is immediate that, for large $\alpha$, neither party wins with probability one at this policy pair, which establishes the claim of non-triviality.

Suppose to the contrary, that for a sequence of $\alpha$’s approaching infinity, $((0, z_L^*), (0, z_R^*))$ is not a PUNE in $G^*_\alpha$. There are two possibilities.

\textit{Case 1.} There is a subsequence of $\alpha$’s such that Left’s militant and opportunist factions would agree to deviate from $(0, z_R^*)$ in the game $G^*_\alpha$. $^{12}$There is a detail here. $A^3*$ only applies if the limit PUNE in the $G^*_\alpha$ game is non-trivial, and the limit of non-trivial PUNE could be a trivial PUNE. Nevertheless, we can deduce that inequality (5.7) will hold for such a limit PUNE, even if it is trivial.
Let, then, \((t_1(\alpha), z_1(\alpha))\) be a policy that Left’s militants and opportunists would agree to deviate to from \((0, z_1^*)\) and that is a best response by Left to \((0, z_1^*)\), in the game \(G_1^*\). It follows that \(z_1(\alpha)\) must be close to \(z_1^*\) for large \(\alpha\), or else the probability of Left victory would be zero, contradicting the supposition that Left’s opportunists agreed to deviate to this point from \((0, z_1^*)\). It therefore follows, by condition (*), that, for large \(\alpha\):

\[
\bar{\mu} - \mu > \frac{(z_1^* - z_1(\alpha))(\mu - w_L)}{2(z_1(\alpha) - a_L)}.
\]  

(Ap.12)

But (Ap.12) plays exactly the role of (5.7): we can invoke the argument in the proof of Proposition 6.1 to conclude that, for large \(\alpha\), \(t_1(\alpha) = 0\), in any Left best response to \((0, z_1^*)\).

Hence Left agrees to deviate to \((0, z_1(\alpha))\) from \((0, z_1^*)\) when facing \((0, z_1^*)\) in \(G_1^*\). But, since both tax rates are zero, this means that Left would agree to deviate from \((0, z_1^*)\) to \((0, z_1(\alpha))\) in the game \(G_1^*\) when facing \((0, z_1^*)\) – which is impossible, since \(((0, z_1^*), (0, z_1^*))\) is a PUNE in \(G_1^*\). The contradiction shows that, for large \(\alpha\), \((0, z_1^*)\) is indeed a best response by Left to \((0, z_1^*)\) in \(G_1^*\).

Case 2. There is a sub-sequence of \(\alpha\)’s such that \((0, z_1^*)\) is not a Right best response to \((0, z_1^*)\) in \(G_1^*\).

Let, then, \((t_2(\alpha), z_2(\alpha))\) be a Right best response in \(G_2^*\) to \((0, z_1^*)\) to which Right’s militants and opportunists agrees to deviate, from \((0, z_1^*)\). We shall similarly prove that, for large \(\alpha\), it must be that \(t_2(\alpha) = 0\), and a contradiction will then follow, just as above. This time, however, we cannot invoke the argument of Proposition 6.1, for we did not study Right’s strategy in that proof. We therefore must prove independently that \(t_2(\alpha) = 0\).

We know, by condition (*), that, for large \(\alpha\):

\[
\bar{\mu} - \mu > \frac{(z_2^* - z_2(\alpha))(\mu - w_R)}{2(z_2(\alpha) - a_R)}.
\]  

(Ap.13)

because if \(z_2(\alpha) - z_2^*\) did not become small, then Left would eventually win with probability one, and \((t_2(\alpha), z_2(\alpha))\) would not be an attractive deviative to Right’s militants from \((0, z_2^*)\) in \(G_2^*\).

Suppose \(t_2(\alpha) > 0\). We shall construct a direction \((-1, \delta(\alpha))\) such that

\[
\nabla_{t_2} s^* \cdot (-1, \delta(\alpha)) < 0
\]  

(Ap.14a)

and

\[
\nabla_{z_2} s^* \cdot (-1, \delta(\alpha)) > 0.
\]  

(Ap.14b)

where the gradients are evaluated at \(((t_2(\alpha), z_2(\alpha)), (0, z_1^*))\), which means that Right would agree to deviate in that direction, in \(G_2^*\).
By differentiating (4.5), we compute that the components of the gradient \( \nabla R s^* \) are given by:

\[
\frac{\partial s^*}{\partial t_R} = \int_w g_{s^*}(w)r\left(\bar{z} + \frac{\Delta t}{\alpha \Delta z} (w - \mu), w\right) \frac{w - \mu}{\alpha \Delta z} \, dw
\]

and

\[
\frac{\partial s^*}{\partial z_R} = \int_w g_{s^*}(w)r\left(\bar{z} + \frac{\Delta t}{\alpha \Delta z} (w - \mu), w\right) \left(1 - \frac{\Delta t(w - \mu)}{\alpha \Delta z^2}\right) \, dw
\]

where, to recall, \( D \) is the denominator in Eq. (Ap.1) or (Ap.4). Using these formulae to expand (Ap.14a), and letting \( \alpha \) tend to infinity, we observe that (Ap.14a) holds for large \( \alpha \) if and only if:

\[
\delta(\alpha) < \frac{2(\mu - \bar{\mu})}{\alpha \Delta z},
\]

(Ap.15)

recalling here that \( \Delta z = z_R(\alpha) - z_L^R \).

Let \( \delta(\alpha) = (2(\mu - \bar{\mu})/\alpha \Delta z) \). Now suppose, contrary to (Ap.14b), that

\[
\nabla R \cdot (-1, \delta(\alpha)) \leq 0.
\]

(Ap.16)

Expanding (Ap.16) yields:

\[
\bar{\mu} - \mu \leq \frac{(z_R(\alpha) - z_L^R)(\mu - w_R)}{2(z_R(\alpha) - a_R)}
\]

which contradicts (Ap.13). Hence (Ap.14b) holds at the above choice for \( \delta(\alpha) \). Consequently, for sufficiently small \( \epsilon \), (Ap.14b) holds for the direction \((-1, \delta(\alpha) - \epsilon) \).

But inequality (Ap.14a) holds as well for the direction \((-1, \delta(\alpha) - \epsilon) \), for any positive \( \epsilon \), since (Ap.15) is true. Hence this case is impossible as well.

References