How Expensive is Commitment?

Oleg Itskhoki

itskhoki@fas.harvard.edu
www.people.fas.harvard.edu/~itskhoki

Preliminary and Incomplete

Macro Lunch
Harvard University

November 15, 2006

Introduction:

Contemporary Macro Models

- Benchmark Neoclassical Model: Representative Agent Stochastic Growth Model (RBC model)
  - Fails empirically on multiple dimensions, especially in the international context (BKK [1992])

Introduction:

Contemporary Macro Models

- Benchmark Neoclassical Model: Representative Agent Stochastic Growth Model (RBC model)
  - Fails empirically on multiple dimensions, especially in the international context (BKK [1992])

- To fix it we add frictions, or augment the benchmark model with constraints:
  - Incomplete markets
  - Transportation costs
  - Sticky prices
  - **Limited Commitment**
  - Informational frictions
  - etc.

Motivation:
Limited Commitment

- Limited Commitment (also Time Inconsistency)
  - very popular friction
  - introduced by Kydland and Prescott [1977]
  - APS [1990] methodology of solving dynamic models with LC

Motivation:

Limited Commitment

- Limited Commitment (also Time Inconsistency)
  - very popular friction
  - introduced by Kydland and Prescott [1977]
  - APS [1990] methodology of solving dynamic models with LC

- Extensive Recent Macro Literature:
  - Endogenously Incomplete Markets:
    - reduces capital flows and the extent of equilibrium risk sharing
    - leads to positively correlated investment
    - Kehoe and Levine [1993], Kocherlakota [1996], Kehoe and Perri [2002]
  - Capital Outflows in Bad States: Atkeson [1991]
  - Fiscal Amplification: Aguiar, Amador and Gopinath [2006]
  - Non-zero Capital Taxes in the Long-Run: Benhabib and Rustichini [1997], Phelan and Stacchetti [2001]

This Paper...

- However often when LC models are simulated they do not provide interesting dynamics in the long-run: commitment problem is fully resolved in the long-run
- Computational papers often need to assume impatience: see Sleet [2006]

This Paper...

- However often when LC models are simulated they do not provide interesting dynamics in the long-run: commitment problem is fully resolved in the long-run
- Computational papers often need to assume impatience: see Sleet [2006]

- I provide a theoretical result about these models: under what conditions LC friction goes away or remains in the long-run:
  - one-sided limited commitment
  - ability to accumulate (risk-free) assets
  - patience: $\beta(1 + r) \geqslant 1$
- The focus of the paper at the moment are SOE models

Main Idea and Results

- Asset accumulation serves the role of collateral for SOE
- This technology is feasible and does not require to waist resources

Main Idea and Results

- Asset accumulation serves the role of collateral for SOE
- This technology is feasible and does not require to waist resources

- It is optimal when patient: $\beta(1 + r) \geqslant 1$
- Moreover, it is nearly costless
- Leads to bounded or unbounded asset accumulation

Main Idea and Results

- Asset accumulation serves the role of collateral for SOE
- This technology is feasible and does not require to waist resources

- It is optimal when patient: $\beta(1 + r) \geqslant 1$
- Moreover, it is nearly costless
- Leads to bounded or unbounded asset accumulation

- Not true when the economy is impatient: commitment problem is never resolved then
- Similar on the transition path for a growing economy

$$\beta \frac{u_{c,t+j}}{u_{c,t}} < \beta$$

Intuition for the Result

- When $\beta(1+r) = 1$ an economy with commitment is indifferent on the margin whether to accumulate assets or not
- If the limited commitment constraint is more tight for low levels of assets, the countries would want to accumulate forever (unless the constraint would no longer ever be binding)
- It is also the intuition for why this accumulation is nearly costless
- Why the economy would not do so when it is impatient even marginally?
  - infeasibility of the first best allocation
  - first order optimality

Related Paper

- Technically, the result is similar to that of Chamberlain and Wilson [1988→2000] and Aiyagari [1994] but is more general (a more general form of the constraint)
  - — Martingale Convergence Theorem for the sequence of the scaled value functions (which in this case form a sub-martingale)

Related Paper

- Technically, the result is similar to that of Chamberlain and Wilson [1988→2000] and Aiyagari [1994] but is more general (a more general form of the constraint)
  - Martingale Convergence Theorem for the sequence of the scaled value functions (which in this case form a sub-martingale)

- Conceptually, the result is similar to back-loading argument in Acemoglu, Golosov and Tsyvinski [2006]:
  - The cost of back-loading is second-order while it provides first order gains from relaxing the IC constraints in all periods prior to the deferred payment
  - Earlier papers on back-loading in principal-agent relationship: Harris and Holmström [1982], Thomas and Worrall [1994], Ray [2002]
  - all are in deterministic set-ups

Implications I

What does this result tell us about the models?

- Many LC models may have interesting predictions but potentially only for off-the-equilibrium dynamics
- LC friction by itself may not be enough
- What is a reasonable additional ingredient?
- When is one-sided LC a reasonable assumption?

Implications II

What does this result tell us about the world?

- Reinforces the Bulow-Rogoff [1989] result: we should expect small countries not only not to borrow but also to save extensively abroad

Implications II

What does this result tell us about the world?

- Reinforces the Bulow-Rogoff [1989] result: we should expect small countries not only not to borrow but also to save extensively abroad
- Why this does not happen?
  - Accumulation technology is imperfect?
  - LC friction is not important?
  - Still on transition path? Development Traps?
  - Impatience?

Outline

Warm-up: CW Result

Consider an endowment SOE characterized by the following Bellman Equation:

$$V(a,z) = \max_{c,a'} \Big\{ u(c) + \beta \mathbb{E}\big\{ V(a',z') \big| z \big\} \Big\}$$

subject to the standard resource constraint:

$$c + a' = (1+r)a + y(z)$$

and the natural debt limit constraint:

$$a' \geqslant \underline{\mathrm{a}}$$

$\longrightarrow$ exogenously incomplete markets

Warm-up: CW Result

- Optimality condition:

$$V_a(a, z) \geqslant \beta(1 + r)\mathbb{E}\{V_a(a', z')\big|z\}$$

with equality when $a' > \underline{\mathrm{a}}$

Warm-up: CW Result

- Optimality condition:

$$V_a(a,z) \geqslant \beta(1+r)\mathbb{E}\{V_a(a',z')\big|z\}$$

with equality when $a' > \underline{\mathrm{a}}$

- Therefore, $\{[\beta(1+r)]^t V_a(a_t,z_t)\}_{t=0}^{\infty}$ is a sub-martingale and by MCT converges (a.s.) since $V_a(\cdot) \geqslant 0$. For $\beta(1+r) \geqslant 1$ this implies convergence of $\{V_a(a_t,z_t)\}_{t=0}^{\infty}$

Warm-up: CW Result

- Optimality condition:

$$V_a(a,z) \geqslant \beta(1+r)\mathbb{E}\{V_a(a',z')\big|z\}$$

  with equality when $a' > \underline{\mathrm{a}}$

- Therefore, $\{[\beta(1+r)]^t V_a(a_t,z_t)\}_{t=0}^{\infty}$ is a sub-martingale and by MCT converges (a.s.) since $V_a(\cdot) \geqslant 0$. For $\beta(1+r) \geqslant 1$ this implies convergence of $\{V_a(a_t,z_t)\}_{t=0}^{\infty}$

- Under some regularity conditions, $u_c(c_t) \propto V_a(a_t,z_t)$, which implies convergence of $\{u_c(c_t)\}_{t=0}^{\infty}$

Warm-up: CW Result

- Optimality condition:

$$V_a(a, z) \geqslant \beta(1+r)\mathbb{E}\{V_a(a', z')\big|z\}$$

  with equality when $a' > \underline{\mathrm{a}}$

- Therefore, $\{[\beta(1+r)]^t V_a(a_t, z_t)\}_{t=0}^{\infty}$ is a sub-martingale and by MCT converges (a.s.) since $V_a(\cdot) \geqslant 0$. For $\beta(1+r) \geqslant 1$ this implies convergence of $\{V_a(a_t, z_t)\}_{t=0}^{\infty}$

- Under some regularity conditions, $u_c(c_t) \propto V_a(a_t, z_t)$, which implies convergence of $\{u_c(c_t)\}_{t=0}^{\infty}$

- Either $c_t \to \bar{c} < \infty$ or $c_t \to \infty$
  - $\to$ The first option is feasible only if $y_t \to \bar{y}$, since $c_t + a_{t+1} = (1+r)a_t + y_t$
  - $\to$ If y_t is mean stationary but does not converge, $a_t \to \infty$
  - $\to$ A problem for SOE models (Schmitt-Grohé and Uribe [2003])

General Set-Up

Bellman Equation:

$$V(a, \eta, z) = \max_{(\xi, a', \eta') \in \Omega} \left\{ u(\xi) + \beta \mathbb{E}\{V(a', \eta', z')|z\} \right\}$$

subject to the technological constraint:

$$a' - (1 + r)a - F(\xi, \eta, \eta'; z) \leqslant 0 \qquad \text{(TC)}$$

and the incentive compatibility constraint

$$\forall z' \in \mathbb{Z} \qquad V(a', \eta', z') \geqslant U(a', \eta', z') \qquad \text{(IC)}$$

- ξ is the control variable
- η is the endogenous state variable other than risk-free assets a
- z is the exogenous state variable
- $U(\cdot)$ is the value after deviation
- $\tilde{\gamma}(z') \equiv \frac{\pi(z')}{1+r} \cdot \gamma(z')$ is LM on (IC) and λ is LM on (TC)

Example

- Utility:

$$u(\xi) \equiv u(c,\ell) = \tfrac{1}{1-\sigma}c^{1-\sigma} - \tfrac{1}{1+\varphi}\ell^{1+\varphi}$$

- Technology:

$$\begin{aligned} F(\xi,\eta,\eta';z) \equiv& F(c,\ell,k,k';z) \\ &= z \cdot f(k,\ell) + (1-\delta)k - k' - c \end{aligned}$$

- Value after Deviation:

$$U(k,z) = \max_{c,\ell,k' \geqslant 0} \Big\{ u(c,\ell) + \beta \mathbb{E}\big\{ U(k',z') \big| z \big\} \Big\}$$

subject to $\quad \widetilde{F}(c,\ell,k,k';z) \geqslant 0$

Assumptions

1. Utility: is increasing in ξ_1 and concave in ξ

2. Technology: $F(\cdot)$ is
   (i) concave in (ξ, η, η')
   (ii) increasing in η and z, decreasing in η'
   (iii) $u_{\xi_j}(\cdot) \cdot F_{\xi_j}(\cdot) \leqslant 0$

3. Deviation Value: increasing and concave in (a, η, z)

4. IC Constraint: $V(a, \eta, z) - U(a, \eta, z) \geqslant 0$ is
   (i) convex in a
   (ii) binding for low enough a
   (iii) slack for high enough a
   (iv) monotone in a: $V_a(\cdot) > U_a(\cdot)$

5. Stationarity: $\{z_t\}$ is mean stationary and has a non-degenerate conditional distributions

Unconstrained Allocation

- Envelop Theorem for asset accumulation:

$$-(1+r)u_{\xi_j}(\xi)/F_{\xi_j}(\xi,\eta,\eta';z) = V_a^*(a,\eta,z)$$

$$(1+r)u_c(c,\ell) = V_a^*(a,k,z)$$

- Optimality condition for asset accumulation:

$$V_a^*(a,\eta,z) = \beta(1+r)\mathbb{E}\{V_a^*(a',\eta',z')|z\}$$

$$u_c(c,\ell) = \beta(1+r)\mathbb{E}\{u_c(c',\ell')|z\}$$

- Optimality Condition for η:

$$\beta\mathbb{E}_t\{\lambda_{t+1}F_{\eta,t+1}\} + \lambda_t F_{\eta',t} = 0$$

$$\beta\mathbb{E}_t\{u_{c,t+1}\cdot[z_{t+1}f_{k,t+1} + (1-\delta)]\} = u_{c,t}$$

Preliminary Results

Lemma 1: Sub-Martingale Property

— Optimal asset accumulation:

$$V_a(a,\eta,z) = \beta(1+r)\mathbb{E}\{V_a(a',\eta',z')|z\}+ \\ +\mathbb{E}\Big\{\underbrace{\gamma(z')}_{\geqslant 0}\cdot\underbrace{[V_a(a',\eta',z')-U_a(a',\eta',z')]}_{>0}\Big|z\Big\}$$

$\rightarrow \; V_a(a,\eta,z) \geqslant \beta(1+r)\mathbb{E}\{V_a(a',\eta',z')|z\}$

Preliminary Results

Lemma 1: Sub-Martingale Property

— Optimal asset accumulation:

$$\begin{aligned} V_a(a,\eta,z) =&\beta(1+r)\mathbb{E}\{V_a(a',\eta',z')|z\}+ \\ &+\mathbb{E}\Big\{\underbrace{\gamma(z')}_{\geqslant 0}\cdot\underbrace{[V_a(a',\eta',z')-U_a(a',\eta',z')]}_{>0}\Big|z\Big\} \end{aligned}$$

$\rightarrow \; V_a(a,\eta,z) \geqslant \beta(1+r)\mathbb{E}\{V_a(a',\eta',z')|z\}$

Result 1: Martingale Convergence Theorem

$$\Big\{[\beta(1+r)]^t V_a(a_t,\eta_t,z_t)\Big\}_{t=0}^{\infty}$$

converges to a non-negative constant along every equilibrium path.

The Main Theorem

$$V_{a,t} = [\beta(1+r)]^{\tau}\mathbb{E}_t V_{a,t+\tau} + \\ + \sum_{j=1}^{\tau}[\beta(1+r)]^{j-1}\mathbb{E}_t\{\gamma_{t+j}\cdot[V_{a,t+j} - U_{a,t+j}]\}$$

The Main Theorem

$$
\begin{aligned}
V_{a,t} =& [\beta(1+r)]^{\tau}\mathbb{E}_t V_{a,t+\tau} + \\
& + \sum_{j=1}^{\tau}[\beta(1+r)]^{j-1}\mathbb{E}_t\left\{\gamma_{t+j}\cdot[V_{a,t+j} - U_{a,t+j}]\right\}
\end{aligned}
$$

Theorem

(a) $\beta(1+r) \geqslant 1$: *the commitment problem is fully resolved and the unconstrained allocation is achieved in the long-run*

- (IC) *is not binding, or* $\gamma_t \to 0$, *as* $t \to \infty$
- $\left[V(a_t,\eta_t,z_t) - V^*(a_t,\eta_t,z_t)\right] \to 0$, *as* $t \to \infty$
- $\eta_t \to \eta_t^*$, *as* $t \to \infty$

The Main Theorem

$$V_{a,t} = [\beta(1+r)]^{\tau}\mathbb{E}_t V_{a,t+\tau} + \\ + \sum_{j=1}^{\tau}[\beta(1+r)]^{j-1}\mathbb{E}_t\{\gamma_{t+j}\cdot[V_{a,t+j} - U_{a,t+j}]\}$$

Theorem

(a) $\beta(1+r) \geqslant 1$: *the commitment problem is fully resolved and the unconstrained allocation is achieved in the long-run*

- (IC) *is not binding, or* $\gamma_t \to 0$, *as* $t \to \infty$
- $\left[V(a_t,\eta_t,z_t) - V^*(a_t,\eta_t,z_t)\right] \to 0$, *as* $t \to \infty$
- $\eta_t \to \eta_t^*$, *as* $t \to \infty$

(b) $\beta(1+r) < 1$: *the commitment problem is never fully resolved and the first best cannot be achieved*

- (IC) *binds, or* $\gamma_{t+\tau} > 0$, *with positive probability*
- $V_t < V_t^*$
- $\eta_{t+\tau} \neq \eta_{t+\tau}^*$ *when* $\gamma_{t+\tau} > 0$, *as long as* $V_{\eta,t+1} \neq U_{\eta,t+1}$.

Additional Results

Proposition

Consider the case $\beta(1+r)=1$. Let $F_{\xi_1} \equiv const$. Then:

(i) ξ_1 *either is eventually a constant or converges to infinity (a.s.)*

(ii) *In the later case, a necessarily converges to infinity as well*

(iii) *In the former case, a remains at the lower bound of the region for which (IC) is not binding and perfect smoothing is feasible*

Additional Results

Proposition

Consider the case $\beta(1+r) = 1$. Let $F_{\xi_1} \equiv const$. Then:

(i) *ξ_1 either is eventually a constant or converges to infinity (a.s.)*

(ii) *In the later case, a necessarily converges to infinity as well*

(iii) *In the former case, a remains at the lower bound of the region for which (IC) is not binding and perfect smoothing is feasible*

Proposition

Consider the case $\beta(1+r) \in (1-\varepsilon, 1)$. For small enough ε, the first best allocation for η is achieved in some states of the world but never in all states of the world.

Remark: This has yet to be proven!

Welfare Costs

Proposition

Consider the case $\beta(1+r) \geqslant 1$.

(a) *If $u_{\xi_1} \equiv const$ (i.e., risk-neutrality), the welfare cost is exactly zero. That is, limited commitment does not reduce welfare.*

Proposition

Consider the case $\beta(1+r) \geqslant 1$.

(a) *If $u_{\xi_1} \equiv const$ (i.e., risk-neutrality), the welfare cost is exactly zero. That is, limited commitment does not reduce welfare.*

(b) *If $u_{\xi_1\xi_1} < 0$ (i.e., risk-aversion), then the welfare costs are positive but second order. That is, it is feasible not to distort the expected present value of the stream of ξ_1, while its allocation across time is distorted.*

Welfare Costs

Proposition

Consider the case $\beta(1+r) \geqslant 1$.

(a) *If $u_{\xi_1} \equiv const$ (i.e., risk-neutrality), the welfare cost is exactly zero. That is, limited commitment does not reduce welfare.*

(b) *If $u_{\xi_1 \xi_1} < 0$ (i.e., risk-aversion), then the welfare costs are positive but second order. That is, it is feasible not to distort the expected present value of the stream of ξ_1, while its allocation across time is distorted.*

Proposition

For the case $\beta(1+r) < 1$, the welfare cost increases as β falls for a given r. The welfare cost is continuous at $\beta(1+r) = 1$. That is, there is a discontinuous change in allocation at $\beta(1+r) = 1$ but the change in welfare cost is continuous.

Next Step...

- Generalize the set-up to a closed economy:
  make r endogenous

- As long as $\beta(1 + r_t)$ converges to a number greater than 1, my results still hold

- The general equilibrium effect of incentive constraints is likely to be increased savings and reduced interest rate (like in Aiyagari [1994])

Applications

Application I: Optimal Capital Taxation without Commitment

Application II: Endogenously Incomplete Markets Model, or Risk-Sharing without Commitment

Application III: Atkeson's [1991] Model: "International Lending with Moral Hazard and Risk of Repudiation"

Application II:

Endogenously Incomplete Markets

- Models of Endogenously Incomplete Markets:
  - Kocherlakota [1996]
  - Kehoe and Perri [2002]

  are models with two-sided lack of commitment

- Models of one-sided lack of commitment are commonly used for SOE's
  - Is this a reasonable assumption?
  - Can this setting arise naturally?

- Lack of commitment reduces the extend of risk-sharing and capital flows

Application II:

Endogenously Incomplete Markets

Utility in Autarky: $U(z) = u(y(z)) + \beta \mathbb{E}\{U(z')|z\}$

Constrained Optimal Value:

$$V(a,z) = \max_{a',c} \left\{ u(c) + \beta \mathbb{E}\{V(a'(z'),z')|z\} \right\}$$

subject to

$$\begin{aligned} \forall z' \in \mathbb{Z} \quad & a'(z') = (1+r)(a-c) + y(z) + d(z), \\ & \mathbb{E}\{d(z')|z\} = 0, \\ \forall z' \in \mathbb{Z} \quad & V(a'(z'),z') \geqslant U(z') \end{aligned}$$

Proposition

(a) *First Best is characterized by perfect consumption smoothing*

(b) *For low a, (IC) cannot be satisfied*

(c) *For high a, (IC) is slack and consumption is smooth*

(d) *For intermediate a, (IC) limits the extent of risk sharing*

(e) *For* $\beta(1 + r) \geqslant 1$ *the economy accumulates assets till it reaches perfect consumption smoothing*

(f) *For* $\beta(1 + r) < 1$*, the economy remains in the region of a for which (IC) has positive conditional probability of being binding and, thus, perfect consumption smoothing is not achieved*

Application II:

Endogenously Incomplete Markets

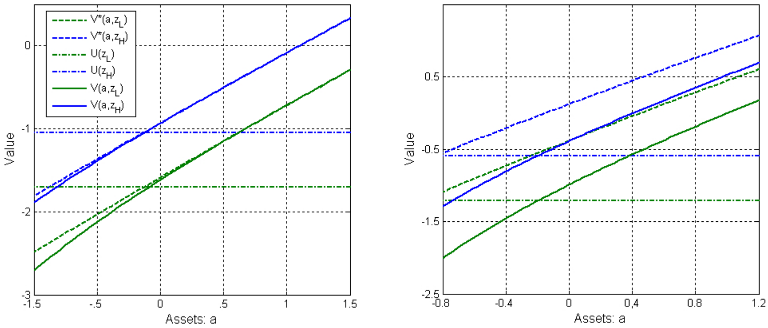

Figure: Value Functions: $\beta(1+r) = 1$ (left) and $\beta(1+r) < 1$ (right)

Application II:
Endogenously Incomplete Markets

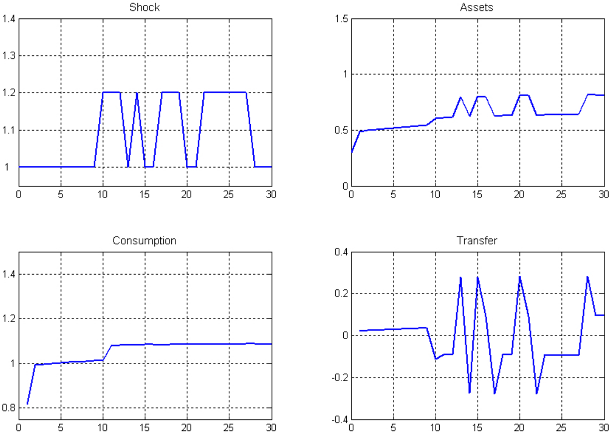

Figure: Equilibrium Dynamics: $\beta(1 + r) = 1$

Application II:

Endogenously Incomplete Markets

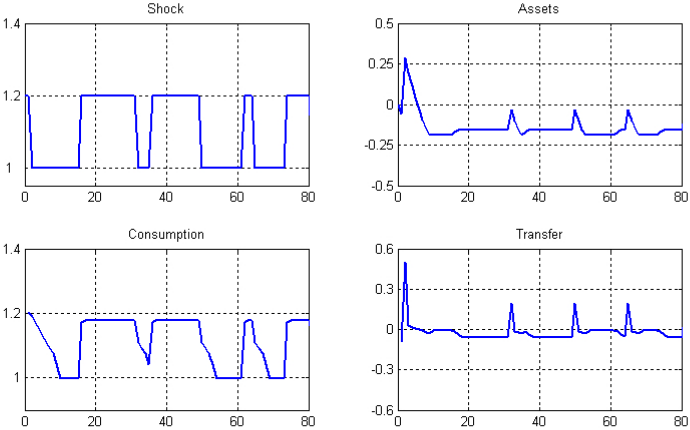

Figure: Equilibrium Dynamics: $\beta(1 + r) < 1$

Application I:
Optimal Capital Taxation

- Ramsey Taxation with Commitment
  - Zero Capital Tax in the Long-Run (Chamley-Judd Result)

Application I:
Optimal Capital Taxation

- Ramsey Taxation with Commitment
  - Zero Capital Tax in the Long-Run (Chamley-Judd Result)
- Non-zero taxes in the Long-Run with the absence of commitment
  - Benhabib and Rustichini [1997]
  - Phelan and Stacchetti [2001]

Application I:
Optimal Capital Taxation

- Ramsey Taxation with Commitment
  - Zero Capital Tax in the Long-Run (Chamley-Judd Result)
- Non-zero taxes in the Long-Run with the absence of commitment
  - Benhabib and Rustichini [1997]
  - Phelan and Stacchetti [2001]
- Zero Taxation Result would be restored if one allows for risk-free asset accumulation
  - Domínguez [2006]
  - Reis [2006]

Application I:
Optimal Capital Taxation

- Ramsey Taxation with Commitment
  - Zero Capital Tax in the Long-Run (Chamley-Judd Result)
- Non-zero taxes in the Long-Run with the absence of commitment
  - Benhabib and Rustichini [1997]
  - Phelan and Stacchetti [2001]
- Zero Taxation Result would be restored if one allows for risk-free asset accumulation
  - Domínguez [2006]
  - Reis [2006]
- Optimal Capital Taxation in a SOE fits my set-up
  - e.g., Aguiar, Amador and Gopinath [2006]

Application III:
Atkeson's [1991] Model

International Lending with Moral Hazard and Risk of Repudiation:

- Stochastic Production Economy with Moral Hazard:

$$Y_t \in \{Y_1, \ldots, Y_N\} \equiv \mathbf{Y}$$

$$g(Y; I) \equiv \Pr\{Y_{t+1} = Y \in \mathbf{Y} | I_t = I\}$$

- Competitive international state contingent lending:

$$b_t = \delta \sum_{Y_{t+1} \in \mathbf{Y}} d_{t+1}(Y_{t+1}) g(Y_{t+1}; I_t)$$

- State variable: Q_t – wealth after repayment on the contract

$$c_t + I_t - b_t \leqslant Q_t \equiv Y_t - d_t(Y_t)$$

- Incentive Compatible Repayment: $V(Q_{t+1}) \geqslant U(Y_{t+1})$

Application III:

Atkeson's [1991] Model

Autarky:

$$b_{t+j} \equiv d_{t+j+1}(\cdot) \equiv 0, \qquad \forall j \geqslant 0$$

$$\Rightarrow \quad U(Q) = \max_{I \in [0,Q]} \left\{ (1-\delta)u(Q-I) + \delta \textstyle\sum_{Y \in \mathbf{Y}} U(Y)g(Y;I) \right\}$$

Complete Markets benchmark:

$$W(Q) = \max_{I \in [0, Q+b], b, d(\cdot)} \Big\{ (1-\delta)u\left(Q - I + b\right) + \\ + \delta \textstyle\sum_{Y \in \mathbf{Y}} W[Y - d(Y)]g(Y;I) \Big\}$$

subject to

$$b = \delta \textstyle\sum_{Y \in \mathbf{Y}} d(Y)g(Y;I)$$

Atkeson Equilibrium:

$$V(Q) = \max_{I,b,d(\cdot)} \Big\{ (1-\delta)u[Q+b-I] + \\ + \delta \textstyle\sum_{Y\in\mathbf{Y}} V[Y-d(Y)]g\left[Y; I(d(\cdot))\right] \Big\},$$

subject to zero profit condition:

$$b = \delta \sum_{Y\in\mathbf{Y}} d(Y)g(Y;I)$$

incentive compatibility of investment:

$$I = \arg\max_{I} \left\{ (1-\delta)u(Q+b-I) + \delta \textstyle\sum_{Y\in\mathbf{Y}} V[Y-d(Y)]g(Y;I) \right\}$$

and incentive compatibility of repayment:

$$V[Y-d(Y)] \geqslant U(Y) \qquad \forall Y \in \mathbf{Y}$$

Capital Outflows

Proposition

If investment choice is internal (not a corner solution) then for low enough Q there would be capital outflows

$$b(Q') < d(Y'|Q) \qquad \textit{for} \qquad Q' = Y' - d(Y'|Q)$$

for the lowest output realization when the no repudiation constraint becomes binding.

Capital Outflows

Proposition

If investment choice is internal (not a corner solution) then for low enough Q there would be capital outflows

$$b(Q') < d(Y'|Q) \qquad \textit{for} \qquad Q' = Y' - d(Y'|Q)$$

for the lowest output realization when the no repudiation constraint becomes binding.

- Remark: However, investment choice is likely to be at the corver for Q very low (especially if $\min\{Y\} \approx 0$) and hence the proposition looses a lot in terms of generality.
- Tsyrennikov [2006]

Application III:
Atkeson's [1991] Model

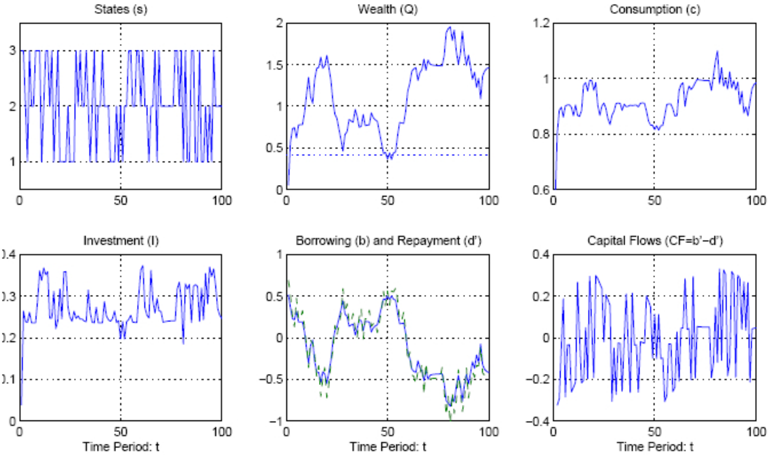

Figure: Equilibrium Dynamics for $\beta(1+r)=1$

Discussion