How Expensive is Commitment?*

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Abstract

In this paper I show, for a general class of models with limited commitment, that an economy is able to effectively buy a commitment technology by accumulating assets that can be seized by its counterparts in case if the economy deviates from the originally prescribed action plan. In fact, this strategy is optimal if the economy is patient enough, i.e. $\beta(1+r) \geqslant 1$, and the welfare costs of this strategy compared to first best are very low. As a result, many of the models, the central feature of which is limited commitment in some form, do not deliver interesting results in the long-run, as the problem of the lack of commitment is entirely resolved by optimal asset accumulation. It is not the case, however, if the economy is impatient, i.e. $\beta(1+r) < 1$, or along the transition path with the growth rates higher than in the long-run. In case of impatience, the limited commitment problem is never resolved fully even in the long-run. The main focus of the paper are small open economy models with endogenously incomplete markets and other frictions.

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1 Introduction

Recently a large number of papers have proposed to incorporate various forms of limited commitment constraints and other similar frictions into the standard macro models to better account for a number of empirical stylized facts. These include reduced capital flows (Atkeson (1991); Kehoe and Perri (2002) and many others), capital outflows after negative shocks (Atkeson, 1991), correlated investment across countries (Kehoe and Perri, 2002), optimality of capital controls (Wright, 2005), fiscal amplification of the real shocks (Aguiar, Amador, and Gopinath, 2006), non-zero capital taxes in the long-run (Phelan and Stacchetti, 2001; Benhabib and Rustichini, 1997), etc.

I argue in this paper that only one friction of this kind is usually not enough to generate interesting dynamics. In particular, this is the case in the models with one sided lack of commitment. In such models, if one allows for asset accumulation (risk-free bond is enough), the country which lacks commitment can always effectively purchase the commitment technology by accumulating assets abroad which can be seized by the rest of the world in the case this country fails to honor its contracts (explicit or implicit). This mechanism essentially means that a country, or an agent, provides collateral as a guarantee that it would honor its contracts.Clearly, this form of purchasing commitment should always be feasible. Moreover, this option arises naturally in most macroeconomic setups. But even when this option is not present in the model exogenously, there is no à priori reason why it would not be created endogenously as there exists a non-trivial demand for such market in equilibrium.

In the paper I show that purchasing commitment is not only feasible but, most importantly, also optimal if the country is patient enough. More precisely, if the country is at least as patient as the rest of the world (i.e., $\beta(1+r) \geqslant 1$, where r is the international risk-free interest rate) it would always want in equilibrium to purchase commitment via asset accumulation. In this case, it is always optimal for the country to accumulate assets all the way to the point where the limited commitment restrictions and other similar frictions are never binding anymore (i.e., at no point along any continuation of the equilibrium path). In some cases it is enough to accumulate a finite amount of assets and already achieve the first best allocation with perfect consumption smoothing (e.g., Aguiar, Amador, and Gopinath (2006) economy; many deterministic setups). In other cases this leads to accumulating assets forever to infinity at a decreasing rate so that in the limit consumption is unbounded (Atkeson, 1991; Chamberlain and Wilson, 2000).

The intuition behind this result is as follows. When $\beta(1+r) = 1$, in the model with commitment and no other frictions, the country is indifferent whether to accumulate or decumulate assets on the margin given the stationary path of output. However, in the model without commitment, asset accumulation has additional value of relaxing the lack of commitment problem (even if only in the probabilistic sense for some not very likely continuations of the equilibrium path). This statement is true for any level of assets for which there remains a possibility (i.e., non-zero probability) of these constraints being binding ever again. Therefore, the country will never stop accumulating assets, even if marginally, unless it reaches the region where it is certain that the frictions are not binding ever again (this region can either be finite or infinite).

When the country is not patient enough (i.e. $\beta(1+r) < 1$), it is not optimal anymore to accumulate assets indefinitely. Moreover, I show that in this case the first best is never achieved and the limited commitment constraints are always binding at least at some states along the equilibrium path. The intuition is straightforward again. Along the first best path the consumption is forever falling. The limited commitment constraints do not allow this to happen in equilibrium and thus the equilibrium is necessarily distorted. The first order optimality logic implies that the constraints should be binding, i.e. that in some states the assets should fall low enough so that the limited commitment constraint becomes effective.

It turns out that buying commitment is not that expensive. When $\beta(1+r) \geqslant 1$ it is always beneficial to lay aside a certain amount of savings which would both pay a market rate of return and serve as a collateral to resolve the commitment problem. The welfare cost of this strategy – *the price of commitment* – can be easily evaluated by comparing the resulting welfare with the welfare under the first best allocation which does not require laying aside any assets. In fact, this strategy is nearly costless. It is exactly costless for the economies with risk-neutral representative agent, while in the economies with risk-averse representative agent the costs of this strategy are second order. As a result, in many model economies the commitment problem is effectively absent in the long-run equilibrium.

Why is this result important? Many models, which use limited commitment arguments, produce interesting predictions which usually allow these models to better fit the empirical stylized facts about capital flows, risk sharing, fiscal and monetary policy, etc. However, current paper shows that these statements can largely be related to the off the equilibrium path dynamics which never or almost never occurs along the equilibrium path.[1] Therefore, despite the fact that these models produce interesting mechanisms that potentially work in the write direction to explain a number of empirical puzzles, effectively they do not introduce significant changes to the equilibrium dynamics compared to the standard macro models which fail empirically in many respects. Therefore, it is crucial to introduce additional features, or frictions,

[1]For example, in the Atkeson (1991) model of "moral hazard and risk of repudiation", capital outflows may occur in the bad states only if the level of assets is very low which would never be the case in the long-run equilibrium provided that the country is patient enough. One would need to assume a significant amount of impatience in this model (way below $\beta = 1/(1+r)$) to produce an empirically relevant frequency of equilibrium capital outflows in the bad states.

which would reinforce or amplify the effects of the limited commitment in these models and thus produce non-trivial changes to equilibrium dynamics. For example, most calibrations of the limited commitment models need to assume impatience ($\beta(1+r) < 1$) to get some action in the equilibrium dynamics.[2]

If one is willing to view this model as an approximate description of the real world, certain policy implications could be made. For example, one of the implications of this paper is that credible international institutions should be created to help small open economies collateralize their commitment problems. Moreover, in a sense, this paper reinforces the Bulow and Rogoff (1989) no borrowing argument. Not only a small country should not borrow in equilibrium, it should want to accumulate assets abroad in the long-run. A few papers have used similar arguments to explain the recent US current account deficit and accumulation of large net foreign asset positions in many Asian countries, notably in China.[3]

Clearly, net foreign asset accumulation is not observed in every small open economy of the world, specifically not in Latin American countries.[4] The model would then suggest that either many small open economies are impatient due to various reasons (e.g., political economy like in Amador (2004)) or current international finance models lack some important market frictions that reverse the logic of the result in the paper.[5] Another possibility is that commitment problems together with other frictions may create impediments to growth for developing economies which lead to development traps with low levels of assets and severe commitment problems over extended periods of time. Finally, the commitment problem in real economies may not be as severe as these models suggest and hence there is no need for long-run asset accumulation.

Links to the literature

Technically, the result here is similar to that in Chamberlain and Wilson (2000) and Aiyagari (1994). The proof of the result uses the Martingale Convergence Theorem for the sequence of value functions, as Envelop Theorem and the Optimality condition assure that the value func-

[2]E.g., see Aguiar, Amador, and Gopinath (2006) extension to the case with debt. Also see references in Sleet (2006).

[3]This argument was pioneered by Michael Dooley in 1990's; see, for example, Dooley, Folkerts-Landau, and Garber (2004). A formalization of this argument can be found in Mendoza, Quadrini, and Ríos-Rull (2007).

[4]As argued in Dooley, Folkerts-Landau, and Garber (2005), Latin American countries are just starting to follow the Asian-countries strategy of reserve accumulation.

[5]One answer is that the assumption of small open economy is inconsistent with unbounded asset accumulation. In particular, this would eventually lead to an increase in the world interest rate. Alternatively, a small open economy may be unable to accumulate large asset positions at the going market rate even when it remains small: e.g., financial market frictions, unavailability of assets to invest in, mutual commitment problems. However, in some setting, like endogenously incomplete markets model, the assumption of the small open economy can remain internally consistent in the long-run (see Section 3.2).

tion scaled by $\beta(1+r)$ is a sub-martingale given the limited commitment constraint. However, I prove the result for much more general "commitment" constraints, while in those papers the constraint had the form of a simple debt limit (endogenous or exogenous).

Conceptually, the result is similar to the back-loading argument as, for example, in Acemoglu, Golosov, and Tsyvinski (2006).[6] In this paper, the payments to the self-interested planner who lacks the ability to commit to policies are optimally back-loaded as long as the planner is as patient as the public. This is essentially equivalent, in our setup, to having country accumulate assets and thus effectively back-load its consumption. The cost of back-loading is second order, while it provides the first order gains from relaxing the incentive constraints at each point on the equilibrium path.[7]

In what follows, we first start by proving our central result in the general setup. Then we discuss a number of popular macro-models which can be viewed as special cases of our general setup. In particular, we analyze the Ramsey model of taxation in the open economy without commitment, the model of international risk-sharing with endogenously incomplete markets, and the Atkeson (1991) model of moral hazard and risk of repudiation.

2 General Theoretical Result

In this section I present my general theoretical result. I start with a simple example from Chamberlain and Wilson (2000) to build up better intuition for the more general theoretical result to follow.

2.1 Warming Up: The Chamberlain-Wilson Example

To develop some intuition, consider the endowment small open economy economy described by the following bellman equation:

$$V(a,z) = \max_{c,a'} \left\{ u(c) + \beta \mathbb{E}\{V(a',z')|z\} \right\}$$

[6]The back-loading argument was first introduced by Harris and Holmström (1982) in the context of the principal-agent relationship. It was also applied in the international lending context by Thomas and Worrall (1994). Finally, the time structure of the self-enforcing agreements was studied by Ray (2002) in the repeated game setting. None of these papers analyze settings with state variables.

[7]The formal proof in Acemoglu, Golosov, and Tsyvinski (2006) relies on the Monotone Convergence Theorem for deterministic sequence of value function which arises in their deterministic framework. My approach is generalized to stochastic frameworks where the sequence of value functions can well be stochastic. In particular, I need to apply Martingale Convergence Theorem, the generalization of the Monotone Convergence Theorem for the stochastic sequences.

subject to the resource constraint

$$c + a' = (1 + r)a + y(z)$$

and the exogenous debt limit

$$a \geqslant \underline{\mathrm{a}}.$$

In words, the economy chooses consumption (c) and assets (a') for the next period given current assets (a) and expectations about the shocks (z) that drive the endowment process ($y(z)$). The choice of the economy is bounded by the standard budget constraint as well as by the debt limit.[8]

The F.O.C. and the E.T. combined yield the following optimality condition on the value function:

$$V_a(a, z) \geqslant \beta(1 + r)\mathbb{E}\{V_a(a', z')|z\}, \tag{1}$$

where $V_a \equiv \partial V/\partial a$ and the inequality holds with equality whenever $a' > \underline{\mathrm{a}}$.

Note that (1) directly implies that the stochastic sequence $\left\{[\beta(1 + r)]^t V_a(a_t, z_t)\right\}_{t=0}^{\infty}$ is a sub-martingale. Moreover, V_a is bounded below by zero. Therefore, we can apply the Martingale Convergence Theorem (Doob, 1953), which states that the stochastic sequence $\left\{[\beta(1 + r)]^t V_a(a_t, z_t)\right\}$ converges almost surely (potentially to some random variable).

For the case $\beta(1 + r) = 1$, this result essentially leaves only two possibilities — either from some point in time t consumption is perfectly smoothed so that $u_c(c_\tau) = V_a(a_\tau, z_\tau) = const > 0$ for all $\tau \geqslant t$, or consumption grows unboundedly and $V_a(a_t, z_t) \to 0$ as $t \to \infty$ (a.s.). In the later case, if one additionally requires some form of stationarity of $\{y(z)\}$, this implies that the country accumulates assets unboundedly ($a_t \to \infty$ a.s.).[9] Finally, if one additionally requires in this economy a non-trivial conditional variance for the endowment process along each possible stochastic path, perfect consumption smoothing can never be achieved for finite levels of consumption.

Why the economy accumulates assets indefinitely when $\beta(1 + r) = 1$? If there were no lower bound on a, the economy would neither accumulate, nor decumulate assets persistently – they would follow a martingale without drift. However, this implies that in finite time the economy would reach any lower bound with probability 1 (given some minor additional technical requirements on the conditional volatility of the endowment process). Therefore, this strategy

[8]For simplicity I assume an exogenous debt limit. Chamberlain and Wilson (2000) show that a debt limit arises endogenously in the incomplete markets setting and is needed to assure the satisfaction of the intertemporal budget constraint, as the sequence of the flow budget constraints is not sufficient in this case.

[9]For a counterexample, when $y(z)$ grows unboundedly in expectation, see problem 16.5 in the Ljungqvist and Sargent (2004) textbook. In that case the assets are stationary at any level while the consumption sequence converges almost surely to infinity together with the stochastic income (endowment) sequence.

is not sustainable when the natural debt limit is present. Without the debt limit, the country is always indifferent between accumulating and decumulating assets on the margin. When the natural debt limit is introduced, accumulating assets also has the additional value of reducing the probability of the debt limit being binding ever in the future. As a result, the country finds it optimal to accumulate assets forever.

We see that under some regularity conditions, in the absence of complete markets[10] the lower bound constraint on assets (upper bound on debt) leads to unbounded asset accumulation along every equilibrium path under very mild conditions given that $\beta(1+r) \geqslant 1$. Moreover, the lower bound in this case arises endogenously. This example is very important as it helps to build intuition for more complicated settings of the later sections. More complicated incentive compatibility constraints effectively create a binding lower bound on the assets and the economy departs from this bound just like in this simple example.

2.2 General Model

I lay out the general setup of the model first. Consider a standard macro-model of an economy which lacks commitments. In particular, think of a small open economy which can trade goods and assets with the rest of the world. Importantly, it is assumed that the rest of the world is able to commit to its policies and honor its contracts and obligations.

The equilibrium dynamics of the economy can be described by the following Bellman equation:

$$V(a,\eta,z) = \max_{(\xi,a',\eta')\in\Omega} \big\{u(\xi) + \beta\mathbb{E}\{V(a',\eta',z')|z\}\big\} \tag{2}$$

subject to the technological constraint[11]

$$a' - (1+r)a - F(\xi,\eta,\eta';z) \leqslant 0 \tag{2$'$}$$

and the incentive compatibility constraint

$$\forall z' \in \mathbb{Z} \qquad V(a',\eta',z') \geqslant U(a',\eta',z'). \tag{2$''$}$$

Here ξ denotes the control variable (vector of size ℓ_1) and η is the endogenous state variable (vector of size ℓ_2) other than risk-free assets a, while z is the exogenous and possibly stochastic state variable (vector of size ℓ_3). Next, $\Omega \subset \mathbb{R}^N$ denotes the space for the triplet (ξ, a, η) and $N = \ell_1 + \ell_2 + 1$. The prime superscript denotes that the variable belongs to the next period.

The incentive compatibility constraint requires that the economy is always better off staying in the contract than breaking it in any state of the world tomorrow. $U(\cdot)$ denotes the value for

[10] Markets are exogenously incomplete in this setup. Contrast it with the example of endogenously incomplete markets in Section 3.2 below.

[11] It could also be a set of constraints where one constraint allows for inter-temporal asset accumulation.

the economy after the deviation. At this stage we take it as exogenous for our analysis. In the specific examples below we obtain this value function endogenously in the model.

The Lagrange multipliers assigned to the constraints are λ and $\tilde{\gamma}(z')$ respectively. Note that λ is always positive (given the non-satiation property of the utility); $\gamma(\cdot)$ is non-negative in general and positive when the incentive constraint binds. For convenience, I normalize the Lagrange multipliers for the incentive constraint in the following way:

$$\tilde{\gamma}(z') = \pi(z')\gamma(z')/(1+r),$$

where $\pi(z')$ is the conditional probability of the exogenous state variable tomorrow given its value today.

We will call the allocation *unconstrained* if the incentive constraint $(2'')$ is not binding. This allocation would be a natural benchmark for the welfare analysis.

The basic example of the technology side is the standard neoclassical growth model. There we have $\xi = (c, l)$, $\eta = k$, and z denoting the productivity shock, so that $\ell_1 = 2$ and $\ell_2 = \ell_3 = 1$. The production technology is given by:

$$F(\xi, \eta, \eta'; z) = zf(k, l) + (1-\delta)k - k' - c,$$

where $f(\cdot)$ is a standard neoclassical production function. We note, however, that this model can nest many other settings. We discuss this in detail in the next section devoted to a number of concrete examples.

Now, we introduce a number of technical assumptions and then discuss them below:

Assumption 1 (Utility) *$u(\cdot)$ maps $\mathbb{R}_+^{\ell_1}$ into $\mathbb{R}$, is concave in all its arguments and increasing in its first argument ξ_1 (which we label* consumption*).*

Assumption 2 (Production Technology) *$F(\cdot)$ maps $\mathbb{R}_+^{\ell_1+2\ell_2+\ell_3}$ into $\mathbb{R}_+$ and is concave, which makes the constraint $(2')$ convex. For concreteness, we assume that $F_\eta(\cdot) > 0$, $F_{\eta'}(\cdot) < 0$ and $F_z(\cdot) > 0$, where the subscripts denote the partial derivatives. For the problem to be well-defined, we require $F_{\xi_j}(\cdot)u_{\xi_j}(\cdot) \leqslant 0$, where ξ_j is the j-th component of ξ. We also assume an Inada condition for η: $\lim_{\eta\to\infty} F_\eta = 0$.*

Assumption 3 (Value after Deviation) *$U(\cdot)$ maps $\mathbb{R}_+^{1+\ell_2+\ell_3}$ into $\mathbb{R}_+$, is concave and increasing in all its arguments (a, η and z).*

Assumption 4 (Incentive Compatibility Constraint) *Incentive Compatibility Constraint $(2'')$ satisfies the following properties:*

(i) it is convex in a: $V_{aa}(\cdot) \geqslant U_{aa}(\cdot)$.

(ii) *for any* (η', z') *there exists* $\underline{a}' = \underline{a}(\eta', z')$ *such that the constraint is binding:* $V(\underline{a}(\eta, z), \eta, z) = U(\underline{a}(\eta, z), \eta, z)$.

(iii) *for any* (η', z') *there exists* $\overline{a}' = \overline{a}(\eta', z')$ *such that for any* (a, η, z) *satisfying* $a > \overline{a}(\eta, z)$ *the constraint is slack.*

(iv) *for all* (η', z') *the equilibrium value function increases in a faster than the value function after deviation:* $V_a(\cdot) > U_a(\cdot)$.

Assumption 5 (Stationarity) *The process for* $\{z_t\}$ *is stationary in some very weak sense. For example, it is mean stationary:* $\mathbb{E}z_t = const$ *for all* t.[12]

Note that the assumptions above together with the standard dynamic programming arguments imply that $V(\cdot)$ is non-negative, concave and increasing in all its arguments. The central result of the paper does not require continuity in any form, but we assume continuity and differentiability of all the functions for simplicity of exposition.

Assumptions 1, 2 and 3 are standard in macroeconomics. They guarantee that the problem is well-behaved (convex programming over bounded sets) and they also have a straightforward economic interpretation.

Assumption 5 is needed to avoid certain undesirable degenerate cases of unbounded income growth as was mentioned in footnote 9. It also excludes the possibility of exogenous economic growth.[13] In this case, as well as on the transitionary path, the intertemporal marginal rate of substitution is

$$\beta \frac{u_{\xi_1,t+1}}{u_{\xi_1,t}} < \beta.$$

Note that it endogenously makes the country effectively less patient than the rest of the world.

Finally, assumption 4 is most important. Intuitively, it means that the incentive constraint is more binding for lower levels of assets a and can be completely relaxed by accumulating enough assets. The convexity of this constraint is the most restrictive assumption, however, as we will see below, it is satisfied in a number of important applications. One special case when it is trivially satisfied is when $U_a(\cdot) = 0$, i.e. $U(\cdot)$ does not depend on assets a. This setting arises naturally when the country after deviation is restricted from the world capital markets (which in many cases can be shown to be the most severe punishment consistent with the ideas of Abreu, Pearce, and Stacchetti (1990)). We take it as our benchmark case, however, it is only

[12]This is a sufficient requirement. The results still hold true even for weaker forms of stationarity of $\{z_t\}$, for example, if $\mathbb{E}_t z_{t+\tau}$ converges to some constant as τ increases.

[13]In fact what matters is that the economy should not persistently grow faster than the rest of the world.

a sufficient requirement for our results and by far not necessary.[14]

The single most important assumption for the result is that the value to the country of staying in the contract $V(\cdot)$ increases in a faster than than its value after deviation $U(\cdot)$. The presence of this feature in the model is driving the back-loading result of asset accumulation. Note that assumption 4 implies that accumulating risk-free assets necessarily alleviates the commitment problem. Hence, there always exists a commitment technology for the country. However, this technology is not costless. Our main result (Theorem 1) below specifies exactly when the country would optimally choose to invest in this commitment technology.

2.2.1 Main Results

The unconstrained allocation (or allocation with commitment) in this setting maximizes (2) subject to (2′) only. We denote the corresponding (first best) value function by $V^*(\cdot)$. The optimality condition central for our analysis is the Euler equation for risk-free asset accumulation:

$$V_a^*(a, \eta, z) = \beta(1 + r)\mathbb{E}\{V_a^*(a', \eta', z')|z\}. \tag{3}$$

Also the following relationship links the marginal utility with the marginal value:

$$-(1 + r)u_{\xi_j}(\xi)/F_{\xi_j}(\xi, \eta, \eta'; z) = V_a^*(a, \eta, z). \tag{4}$$

In the particular example of neoclassical model the last condition reduces to $(1 + r)u_c(c, l) = V_a^*(a, k, z)$ and the standard Euler equation obtains: $u_c(c, l) = \beta(1 + r)\mathbb{E}\{u_c(c', l')|z\}$.

Finally, we are ready to describe the constrained optimum and contrast it with the unconstrained allocation. The Euler equation for risk-free asset accumulation now has the form:

$$V_a(a, \eta, z) = \beta(1 + r)\mathbb{E}\{V_a(a', \eta', z')|z\} + \mathbb{E}\big\{\gamma(z') \cdot [V_a(a', \eta', z') - U_a(a', \eta', z')]|z\big\}, \tag{5}$$

where the second term on the right hand side reflects the possibility of the incentive constraint being binding. This term makes the optimality condition in the model without commitment different from the optimality condition for the unconstrained model.

Lemma 1 (Sub-martingale Property) *Given the assumptions above, in the model with limited commitment the value function scaled by $\beta(1 + r)$ follows a sub-martingale, i.e.*

$$V_a(a, \eta, z) \geqslant \beta(1 + r)\mathbb{E}\{V_a(a', \eta', z')|z\}. \quad \square$$

[14] In particular, introducing lotteries following Prescott and Townsend (1984) is likely to substantially relax the necessary conditions for the main result of the paper. Finally, I note that the results below do not require convexity of the incentive constraint in other state variables (η) which fails in many important examples (e.g., when one allows for capital accumulation after the deviation). Therefore, I do not need to introduce lotteries to convexify this constraint.

Proof. This lemma follows directly from the Euler equation (5), our assumption that $V_a(\cdot) > U_a(\cdot)$ and the non-negativity of the Lagrange multiplier $\gamma(z')$ on the incentive constraint.[15] ■

Lemma 1 together with the non-negativity of the marginal value ($V_a(\cdot) \geqslant 0$) allows us to use the Martingale Convergence Theorem (e.g., see Doob, 1953) to obtain the following result:

Result 1 (Martingale Convergence Theorem) *The random sequence*

$$\left\{[\beta(1+r)]^t V_a(a_t, \eta_t, z_t)\right\}_{t=0}^{\infty}$$

along each equilibrium path (or, equivalently, almost surely) converges to a non-negative constant (potentially different for different equilibrium pathes).

When is (5) different from (3)? To answer this question one can solve forward (5) to get, with some necessary abuse of notation, the following convenient representation:

$$V_{a,t} = \sum_{j=1}^{\tau}[\beta(1+r)]^{j-1}\mathbb{E}_t\left\{\gamma_{t+j} \cdot [V_{a,t+j} - U_{a,t+j}]\right\} + [\beta(1+r)]^{\tau}\mathbb{E}_t V_{a,t+\tau}. \tag{6}$$

Therefore, the solution to the model without commitment will be different from that of the unconstrained model as long as there is a positive probability that along some continuation path the incentive constraint ever binds. This observation allows us to proof our central result:

Theorem 1 *The long-run equilibrium dynamics can be characterized as follows:*

(a) *When $\beta(1+r) \geqslant 1$, the commitment problem is fully resolved and the unconstrained allocation is achieved in the long-run. Formally, this means that the incentive constraint $(2'')$ is not binding asymptotically; γ_t converges to zero and $V(a_t, \eta_t, z_t)$ converges to $V^*(a_t, \eta_t, z_t)$ almost surely; and the choice of the state variable η_t is asymptotically undistorted.*

(b) *When $\beta(1+r) < 1$, the commitment problem is never fully resolved and the unconstrained allocation cannot be achieved. Formally, along each equilibrium path there is always positive probability that the incentive constraint $(2'')$ binds in some future date $t+j$ and $\gamma_{t+j} > 0$. This implies that $V(\cdot) < V^*(\cdot)$ and the choice of the state variable η is necessarily distorted when $\gamma_{t+j} > 0$, as long as in equilibrium $V_\eta(a_t, \eta_t, z_t) \neq U_\eta(a_t, \eta_t, z_t)$.* □

Proof. See Appendix A. ■

Convergence of the scaled marginal value $[\beta(1+r)]^{\tau} V_{a,t+\tau}$ together with representation (6) imply that $[\beta(1+r)]^{\tau-1}\gamma_{t+\tau}[V_{a,t+\tau} - U_{a,t+\tau}]$ converges to zero for all τ as t goes to infinity.

[15]Note that we do not need differentiability to proof a similar result in a more general setting.

When $\beta(1+r) \geqslant 1$, Assumption 4 assures that this implies $\gamma_t \to 0$ and the rest of part (a) of the theorem follows. When $\beta(1+r) < 1$, on opposite γ_t has to be greater than zero with positive probability for any t in order to satisfy condition (6). Part (b) of the theorem follows from this observation.[16]

Theorem 1 is a highly intuitive result. We briefly repeat the intuition behind this result since it was already discussed in some detail in the introduction. Representation (6) demonstrates that a mere possibility of the incentive constraint being binding at some future date creates additional incentives for asset accumulation. Accumulation of assets back-loads the flow of utility with the additional benefit of reducing the incentive problem at all intermediate dates.

When the country is patient enough, i.e. as patient as its international counterparts ($\beta(1+r) \geqslant 1$), it turns out to be optimal to accumulate assets indefinitely until the incentive problem is resolved along any possible continuation of the equilibrium path. When $\beta(1+r) = 1$ the country with a commitment power is indifferent whether to accumulate or decumulate assets on the margin. As a result, the possibility of the commitment constraints being binding creates enough incentives for the country to accumulate additional assets on the margin.

When the country is impatient, i.e. $\beta(1+r) < 1$, the unconstrained allocation with forever increasing marginal value (which corresponds to forever decreasing consumption) can never be incentive compatible. As a result, the country would never accumulate assets indefinitely. On opposite, there would always remain a positive probability for such a country of the incentive constraint being binding in some future date.[17] The less patient is the country (i.e., smaller β for a given r), the higher the probability of the incentive constraints being binding at each date. Also note the discontinuity of the optimal allocation: Even for tiny amounts of impatience ($\beta(1+r) = 1 - \varepsilon$ for arbitrary small $\varepsilon > 0$) the country would choose not to accumulate assets and the incentive constraint would be binding in some periods and states. Finally, binding incentive constraints result in other distortions, in particular, associated with capital accumulation.

More specific characterization of the constrained optimal allocations is given in

Proposition 1 *Let $F(\cdot)$ and $u(\cdot)$ be additively separable in ξ_1. Then, for $\beta(1+r) = 1$, consumption ξ_1 converges (almost surely) either to a constant (perfect smoothing both across time and across states) or to infinity. In the later case, assets (a) necessarily converge to infinity*

[16] We also need to exclude the possibility that $V_{a,t}$ converges to zero in this case. Assume this was the case. Then Assumptions 4 and 5 assure that the incentive constraint is eventually slack while still $V_{a,t} > 0$. However, unconstrained allocation features increasing $V_{a,t}$ at a geometric rate. Therefore, we have a contradiction.

[17] Assume this was not true. Then incentive constraints are never binding and the unconstrained allocation should be attainable which is a contradiction.

as well. In the former case, assets remain at the lower bound of the region for which IC is not binding and perfect smoothing is possible.

Proof. See Appendix B.1. ■

There is a possibility that commitment problem is fully resolved and perfect consumption smoothing is attained for a finite level of assets. As a result, marginal value converges to a positive constant. In particular, this would be the case in the endogenously incomplete markets model of Section 3.2. If commitment problem cannot be fully resolved and perfect consumption smoothing is unattainable for any finite level of assets, asset accumulation continues indefinitely in equilibrium and the marginal value converges to zero. In particular, this would be the case in the moral hazard model of Atkeson (1991) discussed in Section 3.3.

Proposition 1 leads to one important observation: A number of limited commitment models feature unbounded asset accumulation in equilibrium if the country is patient enough. As a result, the assumption of small open economy (together with limited commitment and patience) becomes internally inconsistent. In Section 3.2 we discuss this issue in more detail and provide an example where the assumption of small open economy remains internally consistent in equilibrium.

The assumptions of Proposition 1 can be relaxed considerably in certain cases. In particular, additive separability of $u(\cdot)$ and $F(\cdot)$ in ξ_1 are not needed in the case when perfect consumption smoothing cannot be achieved. To show that ξ_1 and a increase indefinitely in this case we only need the Inada condition on $F_\eta(\cdot)$ and stationarity of the shock (productivity) process (i.e., Assumptions 2 and 5). I discuss this in more detail in Appendix B.1.

2.2.2 Welfare Costs of Limited Commitment

In this section I provide characterization of the welfare costs of limited commitment. Specifically, I define the welfare costs as the difference between the unconstrained value V_0^* and the equilibrium value under limited commitment V_0. Hence, the welfare cost measures exactly the loss of the utility coming from the lack of commitment.[18]

Proposition 2 (Welfare Costs) *Consider the case $\beta(1+r) \geqslant 1$:*

(a) *If the country is risk-neutral (with respect to consumption, ξ_1) and the boundary constraints are not binding (in particular, on ξ_1), then the welfare cost of limited commitment is exactly zero. That is, limited commitment constraint does not reduce welfare.*

[18] If it is optimal for the country to accumulate assets and fully resolve the commitment problem, then this welfare cost can also be called the *price of the commitment.* Alternatively, if the country decides not to resolve the commitment problem fully, then the price of the commitment is too high for the country and it decides not to "purchase" it.

(b) *If the country is risk-averse (with respect to consumption, either across states or across time), then the welfare costs are positive but second order. That is, it is feasible not to distort the expected present value of the steam of consumption* ξ_1*, while its allocation across time is distorted.*

Now consider the case $\beta(1+r) < 1$*. In this case the welfare cost increases as* β *falls for a given* r*. The welfare cost is continuous at* $\beta(1+r) = 1$ *despite the fact that the equilibrium allocation is discontinuous.*

Proof. See Appendix B.2. ■

If the country is risk-neutral with respect to consumption, the allocation of consumption across time and states does not matter as long as the discount factor is taken into account. As a result, the country would be willing to completely back-load all consumption to later periods in order to gain commitment and achieve higher efficiency in production,[19] as long as it can be compensated for waiting (i.e., $\beta(1+r) \geqslant 1$).

When the country is risk averse with respect to consumption, this strategy is still feasible however is likely not to be optimal anymore. The country could have preserved the expected present value of consumption (from unconstrained equilibrium) distorting only the allocation of consumption across time. Therefore, welfare losses in this case are second order, i.e. coming only from the misallocation of consumption across states and time.

The more country is impatient, the higher are the welfare losses since the unconstrained allocation in this case violates the limited commitment constraint sooner and more severely. As a result, the country chooses to have the limited commitment constraint bind more tightly and more often. It is interesting, however, that the welfare loss is continuous at $\beta(1+r) = 1$ despite the sharp discontinuity of the equilibrium allocation at this point (recall Theorem 1). The intuition here is that optimal policy for $\beta(1+r) = 1$ is still feasible and is only marginally inferior as β decreases. This implies that the optimal strategy should lead to even smaller welfare cost and hence the welfare cost increases continuously.

This finishes the exposition of the results in the general setting. Next section illustrates these general theoretical results in a number of popular macroeconomic environments.

3 Applications

In this section we discuss three specific but commonly used models for which our assumptions are satisfied and our theoretical results hold. First, we address the problem of optimal taxation

[19]For example, it may allow the country to commit to low capital tax rates which leads to more efficient levels of capital in equilibrium.

without commitment in a Ramsey framework. There the renowned zero capital tax result under commitment Chamley (1986); Judd (1985) can be restored even when an economy lacks commitment but can accumulate assets abroad.

Next, we look at the optimal risk-sharing problem with one-sided lack of commitment. Arguably this setting naturally arises when a small open economy trades with the rest of the world. In this setting allowing the economy to accumulate risk-free assets abroad allows to restore perfect risk-sharing even under limited commitment.

Finally, we look at the Atkeson (1991) model of moral hazard and risk of repudiation. In that setting as well the commitment problem is fully resolved in the long-run, however, the moral hazard still inhibits perfect risk-sharing.

3.1 Optimal Taxation without Commitment

A number of recent papers have considered optimal Ramsey taxation problems in the economies without commitment (Benhabib and Rustichini, 1997; Phelan and Stacchetti, 2001). The general conclusion from this literature is that zero capital tax result of Chamley and Judd does not survive this generalization as the unconstrained level of capital is often inconsistent with the incentive constraint of the government. However, some recent papers (Domínguez, 2006; Reis, 2006) have challenged these result claiming that once one allows for risk-free asset accumulation the zero capital taxation result can be restored.[20]

In this section I show that my general setup allows to analyze the issues of optimal taxation without commitment in a small open economy model. Indeed, a small open economy by accumulating risk-free assets can effectively resolve its commitment problem and assure zero capital taxes in the long-run.

I use the small open economy model similar to Aguiar, Amador, and Gopinath (2006) but, for simplicity, without shocks to endowment or productivity. The model economy is described by the following Bellman equation:

$$V(a,\tau) = \max_{c,a',\tau'} \left\{ u(c) + \beta V(a',\tau') \right\} \tag{7}$$

subject to

$$\begin{aligned} c + a' &\leqslant (1+r)a + \omega + f\big(k(\tau)\big) - (r+\delta)k(\tau), \\ V(a',\tau') &\geqslant U(\tau'), \end{aligned}$$

where $k(\tau) = \arg\max_k \left\{ (1-\tau)f(k) - (r+\delta)k \right\}$ and $U(\tau) = u\big(\omega + f(k(\tau))\big) + \frac{\beta}{1-\beta}u(\omega)$. In this

[20] The important difference from this work is that all these papers are written for the closed economy framework (see the Discussion section below).

model, a and τ are the endogenous state variables, c is the control variable, and there are no exogenous state variables.

This problem naturally arises in a small open economy settings where all capital belongs to foreigners who directly invest it into the country. Output produced with the capital is subject to a tax rate τ chosen by the small open economy. The citizens of the small open economy consume, invest in the risk-free asset abroad, supply one unit of labor inelastically, and receive endowment of natural resources ω, wages $w = (1-\tau)[f(k) - f'(k)k]$, and tax proceeds $\tau f(k)$.[21]

Though the first best policy is to set $\tau = 0$ (as capital is perfectly mobile before it is invested), the small open economy cannot commit not to tax capital. After the capital is invested the government may well decide not only to tax it but also to seize it completely. If the country does, it looses its net foreign assets as well as all foreign direct investments in all future periods.[22]

Direct examination of problem (7) confirms that it fits the general setup (2) and Assumptions 1–5 are satisfied. Therefore, Theorem 1 and propositions of Section 2.2 hold in this setup. For example, consider the case $\beta(1+r) = 1$. The first best solution in this case is characterized by $\tau^* = 0$, $k^* = \arg\max_k\{f(k) - (r+\delta)k\}$, $c^*(a_0) = ra_0 + \omega + f(k^*) - (r+\delta)k^*$, and $V^*(a_0) = \frac{1}{1-\beta}u\big(c^*(a_0)\big)$. We can compute the threshold $\bar{a}$ from the following condition:

$$V^*\big(c(\bar{a})\big) = U(0) = u\big(\omega + f(k^*)\big) + \frac{1}{1-\beta}u(\omega). \tag{8}$$

When $a > \bar{a}$, the incentive constraint is not binding and the first best allocation is incentive compatible. When $\beta(1+r) = 1$, starting from any initial asset position a_0, the economy will accumulate assets exactly to the level of $\bar{a}$ to attain the unconstrained allocation of capital. If $\beta(1+r) < 1$, the economy will never accumulate assets to the level $\bar{a}$ and always have $\tau > 0$ even in the long run.[23] This results are summarized in

Proposition 3 *If $\beta(1+r) \geqslant 1$, the economy in equilibrium accumulates assets until the commitment problem is fully resolved ($a > \bar{a}$), zero capital taxes ($\tau = 0$) are incentive compatible, and capital is invested efficiently ($k = k^*$). Moreover, if $\beta(1+r) = 1$, the economy will accumulate assets exactly up to the level $\bar{a}$ defined in (8). When $\beta(1+r) < 1$, the economy remains with the level of assets $a < \bar{a}$ in the long run, has positive capital taxes ($\tau > 0$), and there is underinvestment in capital ($k < k^*$).* ■

[21]Combining wages and tax proceeds results in $f(k) - (1-\tau)f'(k)k$. Then using the optimality condition for foreigner ($(1-\tau)f'(k) = r+\delta$), we get the resulting income from the production sector of $f(k) - (r+\delta)k$.

[22]One can show that this is the worst punishment in this case.

[23]Note from (8) that $\bar{a}$ is itself a function of β and r.

3.2 Endogenously Incomplete Markets

In this section I describe the optimal risk-sharing problem with one sided limited commitment, or the endogenously incomplete markets model of a small open economy. This setting naturally arises in the international economics models where a small open economy trades with the rest of the world (ROW). The ROW is assumed to have an effective commitment technology while the small open economy has no commitment ability. In particular, it cannot guarantee that it will pay back its debt to the ROW. The ROW is willing to lend or borrow at the going world risk-free interest rate r and is risk-neutral.

The model economy be described by the following Bellman equation:

$$V(a,z) = \max_{c,a'(\cdot)} \Big\{ u(c) + \beta \mathbb{E}\big\{ V\big(a'(z'), z\big) \big| z \big\} \Big\} \tag{9}$$

subject to

$$\begin{aligned} c' + \textstyle\sum_{z'} \frac{\pi(z'|z)}{1+r} a'(z') &\leqslant a + y(z), \\ V(a'(z'), z) &\geqslant U(z'), \qquad \forall z', \end{aligned}$$

where $q(z'|z) \equiv \frac{\pi(z'|z)}{1+r}$ are the risk-neutral prices of the state contingent bonds $a(z')$ and $y(z)$ is the stochastic endowment process. To show that the budget constraint satisfies the general form of problem (2) we can split the state-contingent bond into two parts — risk free bond and state-contingent assets that are restricted to pay zero on average:

$$a'(z') = \tilde{a}' + d'(z'), \quad \text{with} \quad \textstyle\sum_{z'} \frac{\pi(z'|z)}{1+r} d'(z') = 0.$$

Finally, I define the value after deviation by

$$U(z) = u\big(y(z)\big) + \beta \mathbb{E}\big\{ U(z') \big| z \big\},$$

i.e. once the country defaults on its payments, it is left in autarky and has to consume its endowment without the possibility to share risk.[24]

Again direct examination of problem (9) allows us to conclude that it fits the general setup of problem (2) and all the assumption of Section 2.2 are satisfied. Specifically, consider again the case $\beta(1+r) = 1$. The first best allocation in this case features perfect consumption smoothing

$$c^*(a_0, z_0) = \frac{r}{1+r} \big[Y(z_0) + a_0 \big],$$

where $Y(z_0)$ is the expect present value of endowment defined recursively by

$$Y(z) = y(z) + \textstyle\sum_{z'} \frac{\pi(z'|z_0)}{1+r} Y(z').$$

[24]This assumption can be relaxed so that the country after default is not in autarky but rather can trade only a limited number of assets.

The unconstrained value is given then by $V^*(a_0, z_0) = \frac{1}{1-\beta} u\big(c^*(a_0, z_0)\big)$ and the threshold $\bar{a}(z'|z)$ is defined by

$$V^*\big(c^*\big(\bar{a}(z'|z), z\big)\big) = U(z'). \tag{10}$$

If $a_0 \geqslant \max_{z'} \bar{a}(z'|z_0)$, the incentive constraints are not binding in the next period. Therefore, the economy with limited commitment will accumulate assets to the minimal level such that it comes into every state z with enough assets for the incentive constraints not be binding ever again: $a(z) \geqslant \max_{z'} \bar{a}(z'|z)$. At least one of these inequalities will hold with equality. As a result, full risk sharing is attained in the long run. In the alternative case when $\beta(1+r) < 1$, the country will never accumulate assets to this level and there would remain incomplete risk-sharing at least for some states of the world. These results are summarized in

Proposition 4 *When $\beta(1+r) \geqslant 1$, the economy accumulates assets so that in every state z,*

$$a(z) \geqslant \max_{z'} \bar{a}(z'|z) \tag{11}$$

with $a(z'|z)$ defined by (10)*. This allows the economy to attain perfect risk sharing across states. Moreover, when $\beta(1+r) = 1$, one of the inequalities in* (11) *is satisfied with equality, consumption is equalized both across states and time, and the economy always has a finite net foreign asset position. When beta$(1+r) < 1$, $a(z) < \max_{z'} \bar{a}(z'|z)$ at least in some states z. As a result, perfect consumption smoothing cannot be achieved in some states.* ■

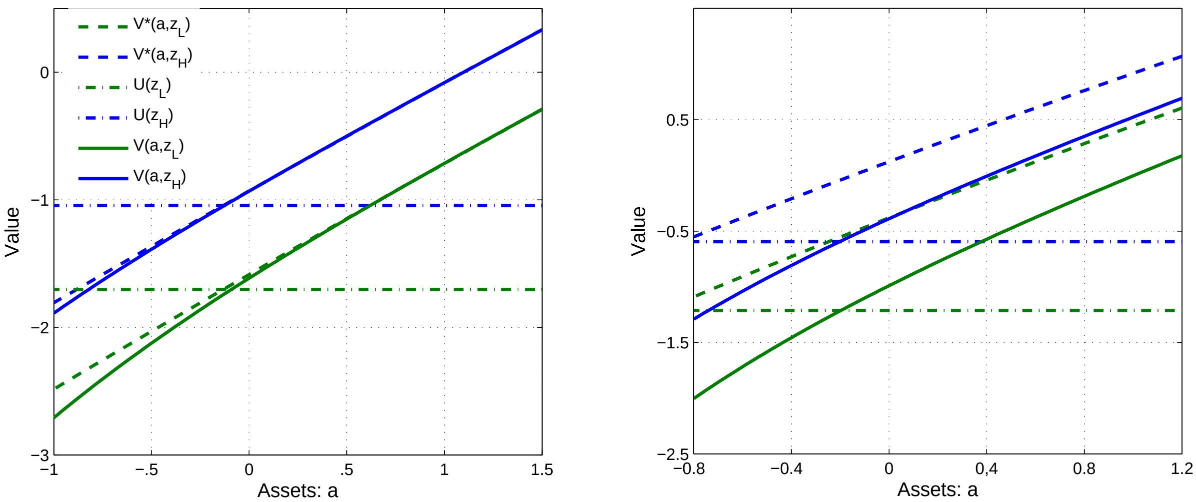

Figure 1: Endogenous Incomplete Markets Model: Value Functions for $\beta(1+r) = 1$ and $\beta(1+r) \leqslant 1$

We illustrate Proposition 4 with a calibration exercise with two states z_L and $z_H > z_L$. The results are presented in Figures 1–3. Figure 1 plots the value functions (first best, autarky and limited commitment) for different levels of assets and realizations of the shock.

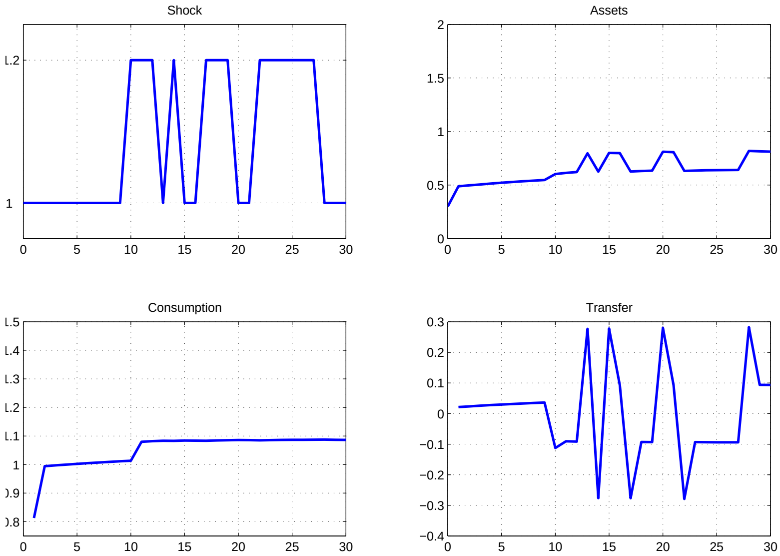

Figure 2: Endogenous Incomplete Markets Model: Dynamic Path when $\beta(1 + r) = 1$

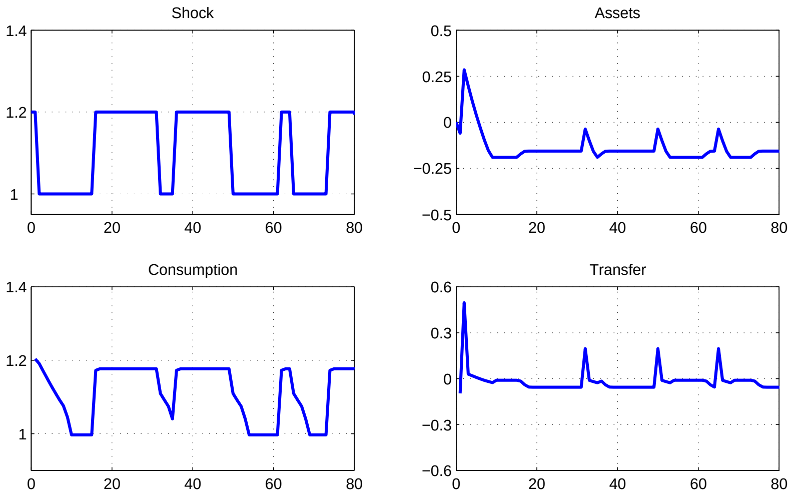

Figure 3: Endogenous Incomplete Markets Model: Dynamic Path when $\beta(1 + r) = 1$

Figures 2 and 3 plot typical equilibrium dynamic for $\beta(1+r) = 1$ and $\beta(1+r) < 1$. Note the pattern of capital flows. For $\beta(1 + r) = 1$ they are first restricted, when the country start with low level assets and the limited commitment constraint is still binding, and then unrestricted as the country accumulates enough assets and reaches the region of perfect consumption smoothing. For the case $\beta(1+r) < 1$ the opposite is true. At first the country has large capital movements while the limited commitment constraint is not very binding as it starts out with a lot of assets, but over time it decumulates assets to the region where the limited commitment constraint is tighter and hence the capital flows are more restricted. As argued in Introduction, limited commitment constraint in this setting leads to non-trivial long-run dynamics only under the assumption of impatience ($\beta(1+r) < 1$).

3.3 Atkeson Economy: Limited Commitment and Moral Hazard

The final application we consider is the Atkeson (1991) model of the risk of repudiation and moral hazard. In this setting not only the country cannot commit to pay its debt, it also has a moral hazard friction in investment. The strong result in this setting due to Atkeson (1991) is that at least sometimes the low realizations of output are associated with capital outflows which contradicts the standard logic of risk-sharing in the models without moral hazard.

It turns out that our main argument extends to this setting as well. The difference of this setting from the simple risk-sharing problem is that the moral hazard in investment inhibits perfect risk-sharing even when incentive constraint for debt repayment is not binding. As a result first best is never achieved and assets are accumulated indefinitely, as there always exists positive probability of a very long sequence of bad output realizations which would lead the economy to the state where incentive constraints bind. Accumulation of assets alleviates significantly both frictions in the economy. As a result, even in this setting, capital outflows in the bad states never occurs in the long-run and the correlation of output with the capital flows is essentially negative 1.

Formally, the model can be described by the following Bellman equation:[25]

$$V(a,z) = \max_{c,I,a'(\cdot)} \left\{ u(c) + \beta \sum_{z'} \pi(z') V(a'(z'), z') \right\} \tag{12}$$

subject to

$$\begin{aligned} c + I(b,Q,d') - b &\leqslant y(z) + a(z) \equiv Q(z), \quad \text{where} \quad b = -\frac{1}{1+r}\sum_{z'} \pi(z')a'(z'), \\ V(a',z') &\geqslant U(z'), \quad \forall z'. \end{aligned}$$

There are two important differences of this setup from the basic endogenous incomplete markets model considered in the previous section. First of all, the level of investment affects the

[25]The only notational difference here from Atkeson's original paper is that he used $d(z) = -a(z)$ and called it repayment. Also b is called borrowing, while I use state-contingent assets (a) as the state variable. Finally, Atkeson shows that $Q(z) = y(z) + a(z)$ is a sufficient state variable. To be consistent with the previous results, I use the pair (a, z) as the state vector.

probability of the states next period so that $\pi(z') = g(z'; I)$; higher level of investment shifts the probability mass towards higher realizations of output. Secondly there is a moral hazard problem since the country cannot commit to invest optimally. As a result, there is an incentive compatibility constraint which determines

$$I(b, Q, d') = \max_{I \geqslant 0} \left\{ u\big(Q - b - I\big) + \beta \sum_{z'} g(z'; I) V\big(a'(z'), z'\big) \right\}. \tag{13}$$

Note that this incentive constraint takes the level of b as given. Therefore, the country without commitment cannot internalize the effect of lower I on the higher price of borrowing for it which works through $g(z'; I)$. Note that (13) makes it clear that perfect risk-sharing (which implies constant V across all states) is incentive compatible only with zero level of investment. Finally, we define the value of deviation as

$$U(z) = \max_{I \in [0, y(z)]} \left\{ u\big(y(z) - I\big) + \beta \sum_{z'} g(z'; I) U(z') \right\}.$$

Again direct inspection of the problem (12) makes it clear that it satisfies the general setup and assumptions of Section 2.2. Since perfect consumption smoothing is not possible as long as equilibrium features positive level of investment, this model will necessarily predict unbounded asset accumulation and consumption growth in the long run when $\beta(1+r) = 1$, just like in the Chamberline-Wilson example.[26] We summarize this in

Proposition 5 *In Atkeson model when $\beta(1 + r) = 1$ and as long as investment is positive in equilibrium (i.e., internal solution), perfect consumption smoothing is never achieved, assets are accumulated unboundedly, consumption increases unboundedly, and the only friction remaining in the long-run is moral hazard in investment, i.e. the limited commitment incentive constraint is slack. As a result, long-run equilibrium dynamics features no capital outflows in the bad states.*

This result is illustrated in Figure 4 which plots the typical dynamic path of the Atkeson economy (with three possible states: $z \in \{z_L, z_M, z_H\}$). The economy starts in the region with low assets a_0 and wealth Q_0. In this region the incentive problem is severe and investment is depressed as a result. However, then economy accumulates assets fast to the high end of the grid ($Q = 2$ in this simulation)[27] where the incentive problem is not present. Capital flows are large (of the same magnitude as the fluctuations in output; for the present parametrization $y \in \{0.75, 1, 1.25\}$) and highly negatively correlated with output (the correlation is above 0.95 in absolute value) as illustrated in Figure 5.

[26]This is clearly inconsistent with the assumption of small open economy.

[27]This is an obvious limitation of the numerical methods in dealing with models that predict an unbounded growth of state variables.

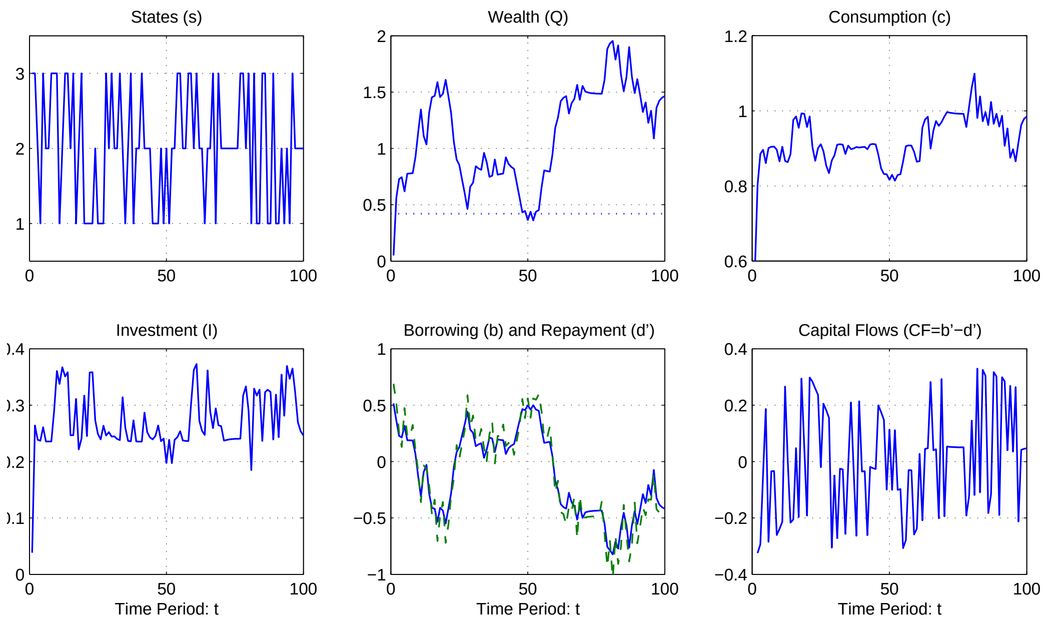

Figure 4: Atkeson Model: Dynamic Path when $\beta(1+r) = 1$

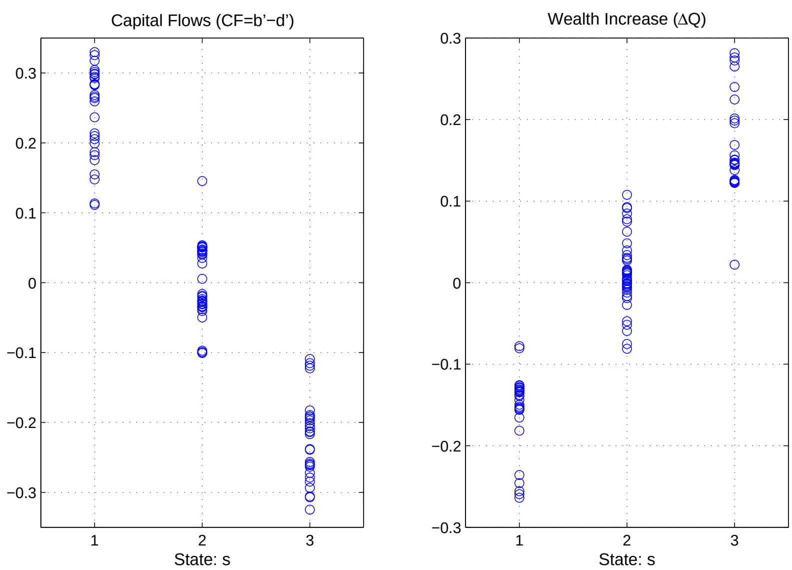

Figure 5: Atkeson model: Capital Flows and Wealth Accumulation

4 Concluding Remarks

In this paper I show that for a general class of small open economy models with limited commitment the country is effectively able to buy a commitment technology by accumulating assets abroad. Net foreign assets play the role of collateral which can be seized in case if country deviates from the explicit or implicit contract with the rest of the world. I also show that this is in fact the equilibrium outcome when the country is at least as patient as the rest of the world (i.e., $\beta(1+r) \geqslant 1$). Therefore, models with limited commitment that do not assume impatience at the same time feature no commitment problem in the long-run equilibrium.

The results of the paper are likely not to generalize to closed economy settings and to open economy setting with mutual commitment problems. In the closed economy there is no outside body which can credibly seize the assets of the government once it deviates. When there is multilateral limited commitment, asset accumulation by all countries is impossible in equilibrium and the interest rate will be endogenously driven down in equilibrium to levels that guarantee $\beta(1+r) < 1$ for most countries (see Aiyagari (1994)). A natural general equilibrium setup in which this analysis is likely to hold through is the one in which a large economy is trading with a large number of small economies. The model will predict in this case that all small open economies accumulate assets of the large open economy till their commitment problems are resolved. In fact, this setup can be valid for the analysis of many empirical issues in the global economy such as recent accumulation of U.S. assets by a number of developing countries.

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A Proof of the Theorem 1

The first order condition for asset accumulation (5) implies for some distant date in the future that

$$[\beta(1+r)]^{\tau}\mathbb{E}_t V_{a,t+\tau} = [\beta(1+r)]^{\tau+1}\mathbb{E}_t V_{a,t+\tau+1} + [\beta(1+r)]^{\tau}\mathbb{E}_t\big\{\gamma_{t+\tau+1}\cdot[V_{a,t+\tau+1}-U_{a,t+\tau+1}]\big\}.$$

The convergence of the sequence $\{[\beta(1+r)]^{\tau}V_{a,t+\tau}\}$ along each equilibrium path and the non-negativity of $\gamma_{t+\tau+1}\cdot[V_{a,t+\tau+1}-U_{a,t+\tau+1}]$ imply the convergence to zero of

$$[\beta(1+r)]^{\tau}\cdot\gamma_{t+\tau+1}\cdot[V_{a,t+\tau+1}-U_{a,t+\tau+1}]\to 0 \qquad \text{as} \quad \tau\to\infty$$

along every equilibrium path.[28]

(a) Consider first the case of $\beta(1+r)\geqslant 1$. In this case, the result above implies the convergence to zero of the sequence $\{\gamma_{t+\tau+1}\cdot[V_{a,t+\tau+1}-U_{a,t+\tau+1}]\}$. First, note that for any finite level of $(a_{t+\tau+1},\eta_{t+\tau+1},z_{t+\tau+1})$, $V_{a,t+\tau+1}>U_{a,t+\tau+1}\geqslant 0$.[29] Now, if $a_{t+\tau+1}$ converges to infinity, we may have $V_{a,t+\tau+1}$ and $U_{a,t+\tau+1}$ converging to zero, but by Assumption 4 the incentive constraint will become eventually slack as $a_{t+\tau+1}$ increases unboundedly. Therefore, indeed

$$\gamma_{t+\tau+1}\to 0 \qquad \text{as} \quad \tau\to\infty \quad (a.s.),$$

the incentive constraint ($2''$) is (almost surely) asymptotically slack and the equilibrium allocation is asymptotically undistorted.

Now consider the optimality condition for the choice of the state variable η. Combining together the F.O.C. and the E.T. for η, we obtain:

$$\beta\mathbb{E}_t\{\lambda_{t+1}F_{\eta,t+1}\}+\lambda_t F_{\eta',t}+\mathbb{E}_t\{\gamma_{t+1}\cdot[V_{\eta,t+1}-U_{\eta,t+1}]\}=0. \tag{14}$$

Since $\{\gamma_t\}$ converges asymptotically to zero, we have

$$\big[\beta\mathbb{E}_t\{\lambda_{t+1}F_{\eta,t+1}\}+\lambda_t F_{\eta',t}\big]\to 0 \qquad \text{as} \quad t\to\infty \quad (a.s.)$$

and hence the choice of η is asymptotically undistorted.[30] Note that for the particular case of the neoclassical setup this implies that the standard Euler equation for capital holds asymptotically:

$$\beta\mathbb{E}_t\{u_{c,t+1}\cdot[z_{t+1}f_{k,t+1}+(1-\delta)]\}=u_{c,t}.$$

[28] All convergence here are with probability 1 (or almost surely).

[29] In equilibrium, $z_{t+\tau+1}$ is finite by the Assumption 5 of stationarity and $\eta_{t+\tau+1}$ is finite due to Inada condition (see foonote 30).

[30] Note that for the choice of η to be bounded it is sufficient to require that $F_\eta\to 0$ as $\eta\to\infty$ while $F_{\eta'}<0$ for all η'. Also note that $(1+r)\lambda_t=V_{a,t}$ from the envelop theorem. Therefore, we have

$$\mathbb{E}_t\{F_{\eta,t+1}/(-F_{\eta',t})\}\to(1+r) \qquad \text{as} \quad t\to\infty \quad (a.s.),$$

which is exactly the condition for unconstrained allocation of the state variable.

Finally, note that asymptotically the choice of $(\xi_t, \eta_{t+1}, a_{t+1})$ is undistorted (as γ_{t+1} goes to zero) which implies that

$$\left[V(a_{t+1}, \eta_{t+1}, z_{t+1}) - V^*(a_{t+1}, \eta_{t+1}, z_{t+1})\right] \to 0 \qquad \text{as} \quad t \to \infty \quad (a.s.)$$

Note that this does not imply the convergence of the value functions, it only implies the convergence of the sequence of value functions evaluated at the particular equilibrium sequence of the variables $\{(a_{t+1}, \eta_{t+1}, z_{t+1})\}$.This finishes the proof of this part of the theorem.

(b) Now consider the case of $\beta(1+r) < 1$. There are two possibilities to consider: (i) $V_{a,t} \to 0$ as $t \to \infty$; and (ii) $V_{a,t+j} \geqslant \delta > 0$ with a positive probability for every t and some $j > 0$ and for some $\delta > 0$. We consider these two possibilities in turn:

(i) Let $V_{a,t} \to 0$. The Inada condition assures the finiteness of η and the stationarity assumption 5 assures the finiteness of z. As a result, $V_{a,t}$ can converge to 0 only if a_t converges to infinity. But this necessarily implies $\gamma_t = 0$ once a_t reaches some finite level $\bar{a}$ (using Assumption 4). Hence, when a_t is large enough but less then infinity, the Euler equation can be as

$$V_{a,t} = \beta(1+r)\mathbb{E}_t V_{a,t+1}$$

which implies

$$V_{a,t+1} = \frac{1}{\beta(1+r)} V_{a,t} > V_{a,t} > 0$$

and, therefore, is inconsistent with the convergence of $V_{a,t}$ to zero. To put it in words, as the country will accumulate enough assets, $V_{a,t}$ will necessarily reflect from the environ of zero. Therefore, for any t there exists $\tau > 0$ and $\delta > 0$ such that $V_{a,t+\tau} \geqslant \delta$ with a positive probability which brings us to the next case.

(ii) Consider two dates t_1 and $t_2 > t_1$ at which $V_{a,t_i} \geqslant \delta$ and $t_2 - t_1$ is large enough. Then we have

$$V_{a,t_1} - \left[\beta(1+r)\right]^{t_2-t_1}\mathbb{E}_{t_1} V_{a,t_2} = \textstyle\sum_{\tau=t_1+1}^{t_2}\left[\beta(1+r)\right]^{\tau-t_1}\mathbb{E}_{t_1}\left\{\gamma_\tau \cdot \left[V_{a,\tau} - U_{a,\tau}\right]\right\} \geqslant 0.$$

Note that $[\beta(1+r)]^{t_2-t_1}$ can be made arbitrary small by making $t_2 - t_1$ large enough. Therefore, both sides of this expression can be made strictly positive which necessarily implies that there is a positive probability of $\gamma_\tau > 0$ for $\tau \in \{t_1+1, \ldots, t_2\}$.

This proves that along each equilibrium path there is a positive probability that $\gamma_{t+\tau} > 0$ and the incentive constraint binds. In turn, this immediately implies that $V(a, \eta, z) < V^*(a, \eta, z)$ for any (a, η, z). Finally, the optimality condition for the choice of η is necessarily distorted whenever $\gamma_{t+j} > 0$, as long as $V_\eta(\cdot) \neq U_\eta(\cdot)$ for $(a_{t+j}, \eta_{t+j}, z_{t+j})$. This finishes the proof of the theorem. ■

B Other Proofs

B.1 Proof of Proposition 1

Without assuming separability, the first order condition for the choice of consumption implies $-(1+r)u_{\xi_1,t}/F_{\xi_1,t} = V_{a,t}$. Therefore, $[\beta(1+r)]^\tau(u_{\xi_1,t+\tau}/F_{\xi_1,t+\tau})$ also converges along each equilibrium path. When $\beta(1+r) \geqslant 1$, this, together with the separability in ξ_1 assumption, implies the convergence of $\xi_{1,t}$ along each equilibrium path.

If $V_{a,t}$ converges to a positive number, then $\xi_{1,t}$ converges to a finite number, i.e. perfect consumption smoothing. However, once perfect consumption smoothing is attained, there is no additional incentive for accumulating assets. Therefore, assets always remain at the lower bound of the region for which perfect consumption smoothing is incentive compatible.

Alternatively, if $V_{a,t}$ converges to zero, then $\xi_{1,t}$ converges to infinity. However, with a stationary process for z_t (Assumption 5) and Inada conditions for η_t (Assumption 2), this can be possible only if a_t also converges to infinity.

Now relax the separability assumption. Then, assuming that $F_{\xi_1}(\cdot)$ is bounded, the convergence of $V_{a,t}$ to 0 still implies the convergence of u_{ξ_1} to zero, which can happen only if ξ_1 converges to infinity. Therefore, in this case the assumption of separability is not crucial. ■

B.2 Proof of Proposition 2 (a sketch)

If the country is risk-neutral with respect to ξ_1, it can relocate it freely across periods as long as it is compensated for time discounting. In particular, it can accumulate enough assets in the early period (as long as the boundary condition on ξ_1 is not binding), so that the incentive constraint ($2''$) is slack starting next period on. When $\beta(1+r) \geqslant 1$, the market rate of return on a would (at least) compensate the country for delaying consumption. Therefore, this strategy has no welfare costs.[31]

When the country is risk-averse with respect to ξ_1, the same action plan is still feasible but may not be optimal anymore. Therefore, equilibrium utility is no less than the utility after following the above action plan. Next, note that the above action plan weakly increases the present value of consumption if β is taken as the discount factor (and $\beta(1+r) \geqslant 1$). Therefore, the utility under this action plan is decreased only due to the intertemporal allocation of consumption and not due to the decrease in the average level of consumption. Finally, since

[31] If the boundary condition on ξ_1 is binding, this result still may go through as long as β is large enough and the benefits of attaining optimal allocation in the future outweigh the commitment problem today. In this case, the accumulation phase will take a number of periods, however, capital will be at its unconstrained level already in the first period.

the welfare loss under the constrained optimal allocation is even smaller, the loss from limited commitment is second order.

Now consider the case of $\beta(1+r) < 1$. Evaluate the utility from following the optimal action plan under $\beta = 1/(1+r)$. This action plan is still feasible and the deviation from optimality is continuous in β. Therefore, since constrained optimal value is at least as large, the welfare cost is continuous at $\beta(1+r) = 1$. Finally, recall that smaller β for a given r implies that the incentive constraint in equilibrium is more tight (both tight more often and more tight when it is tight). Therefore, the welfare cost is higher for lower β. ■