A model of time-varying risk premia with habits and production

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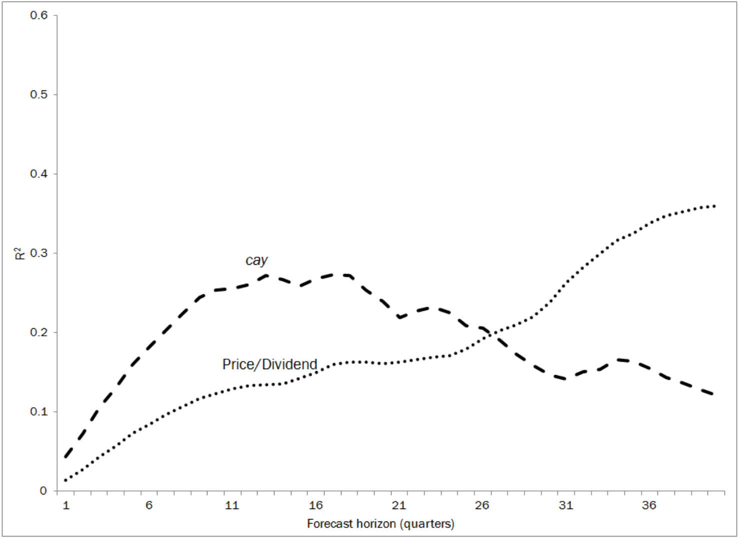
Introduction

- Aggregate stock returns are highly volatile
  - More than can be explained by dividends by a factor of 5 (Shiller, 1981; LeRoy and Porter, 1981)
  - Implies discount rates vary
  - Point estimates suggest returns are predictable

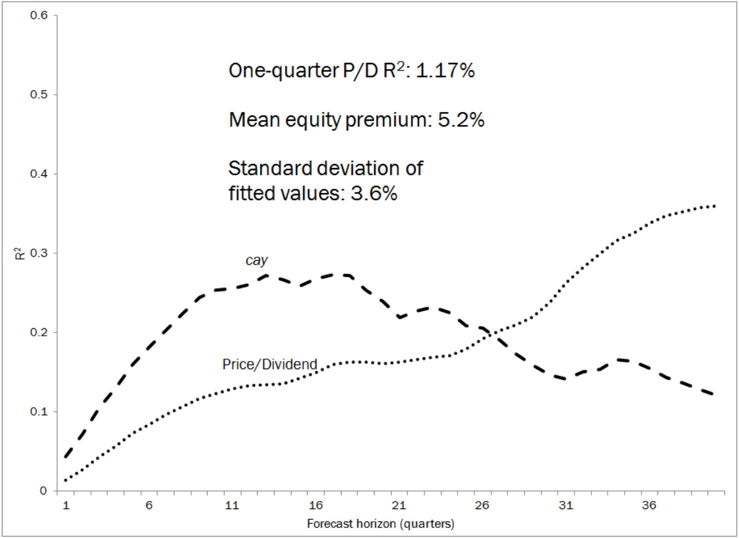
Predictability evidence

- Numerous predictors of stock returns
  - P/E, P/D ratios (Fama and French, 1988; Campbell and Shiller, 1988), Consumption/wealth ratio (Lettau and Ludvigson, 2001)
  - Interest rates (Fama and French, 1989)
- Predictability is pervasive
  - Corporate debt; US treasuries; foreign sovereign debt; commodity returns
- Statistical inference is difficult...
- Expected returns seem high in recessions

Stock return predictability



Stock return predictability



Introduction

- Both price and quantity of risk vary over time
- Time-varying risk aversion usually from habits
  - Campbell and Cochrane (1999); Lettau and Uhlig (2000)
  - Based on power utility $\Rightarrow$ changing risk aversion affects intertemporal substitution
  - Unrealistic endogenous consumption, interest rate dynamics
  - Long-run risks are unpriced
  - Consumption should strongly forecast returns

plot

What I do

- Combine Campbell–Cochrane with Epstein–Zin
  - Risk aversion low in good times, separate from intertemporal subs.
- Can match standard macro and finance moments
  - Short and long-run variances of output, consumption, investment
  - Mean and variance of Sharpe ratio, risk-free rate
- Can estimate risk aversion and forecast returns

GE models with time-varying expected returns

- Kaltenbrunner and Lochstoer (2010): stochastic volatility
  - But both quantity and price of risk are countercyclical plot
- Campanale, Castro, and Clementi (2010): first-order risk aversion
- Guvenen (2009), De Graeve at al. (2010): limited participation
- Chan and Kogan (2002), Xiouros and Zapatero (2010): heterogeneous risk aversion
- Lettau and Uhlig (2000): Campbell–Cochrane habits with production
- Gourio (2010): time-varying disaster risk; matches empirical predictability

Outline

1. Preference specification
2. Production economy, solution
3. Estimate of risk aversion
4. EIS estimates
5. Extensions

Household preferences

- Time-separable utility:

$$V_t = \frac{C_t^{1-\rho}}{1-\rho} + \beta E_t V_{t+1}$$
$$= E_t \sum_{j=0}^{\infty} \beta^j \frac{C_{t+j}^{1-\rho}}{1-\rho}$$

- Epstein-Zin preferences:

$$V_t = \left\{ (1-\beta) C_t^{1-\rho} + \beta G^{-1} \left(E_t \left[G \left(V_{t+1} \right) \right] \right)^{1-\rho} \right\}^{1/(1-\rho)}$$

  - Risk aversion determined by G
  - Without uncertainty, G has no effects

- CRRA (Weil, 1989; Epstein and Zin, 1991):

$$
\begin{aligned}
G_t^{CRRA}\left(V_{t+1}\right) &= V_{t+1}^{1-\alpha} \\
V_t &= \left\{(1-\beta)\, C_t^{1-\rho} + \beta E_t \left[V_{t+1}^{1-\alpha}\right]^{\frac{1-\rho}{1-\alpha}}\right\}^{1/(1-\rho)}
\end{aligned}
$$

- Habit specification:

$$G_t^{Habit}\left(V_{t+1}; H_t\right) = \left(V_{t+1} - H_t\right)^{1-\alpha}$$

  - RRA is low when V_{t+1} is close to habit

$$RRA = \frac{V_{t+1}}{V_{t+1} - H_t}\alpha$$

  - I assume H_t is *external* to the household
- Agents care about value relative to an external benchmark
  - Habit puts structure on changes in risk aversion
  - Not an addiction or subsistence level
  - Testable at the household level

Evidence on habits

- Carroll (2002): high wealth associated with high risky portfolio share
- Calvet et al. (2009), Calvet and Sodini (2010): pos. wealth shocks increase risky share
    - Brunnermeier and Nagel (2008): portfolios mostly driven by intertia
- Bucciol and Miniaci (2011), Guiso et al. (2011): households became more risk averse following recessions
- Tanaka et al. (2010): pos. wealth shocks reduce loss aversion
- Thaler and Johnson (1990); Gertner (1993); Post et al. (2008): house money effect

Alternatively: put habit in consumption

$$V_t = \left\{ (1-\beta)\left(C_t - H_t\right)^{1-\rho} + \beta E_t \left[V_{t+1}^{1-\alpha} \right]^{\frac{1-\rho}{1-\alpha}} \right\}^{1/(1-\rho)}$$

- Same problems as Campbell–Cochrane:
  - Habit affects intertemporal substitution
  - Cosumption should forecast returns (it doesn't)
  - Long-run risks are not priced
- Weak micro evidence

$$V_t = \left\{ \begin{array}{c} (1-\beta)\, C_t^{1-\rho} \\ +\beta \left[\left[E_t \left(V_{t+1} - H_t \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}} + H_t \right]^{1-\rho} \end{array} \right\}^{\frac{1}{1-\rho}}$$

- Standard Epstein–Zin results – long-run risks matter; risk aversion is separate from the EIS
- Three problems:
  - Tractability
  - Existence of integrals
  - Representative agent

Equivalence

$$
\begin{aligned}
\text{Certainty equivalent} \quad &: \quad G^{-1}\left\{E_t\left[G\left(V_{t+1}\right)\right]\right\} \\
\left(E_t\left[\left(V_{t+1}-H_t\right)^{1-\zeta}\right]\right)^{\frac{1}{1-\zeta}}+H_t \quad &\approx \quad \left(E_t\left[V_{t+1}^{1-\alpha_t}\right]\right)^{\frac{1}{1-\alpha_t}} \\
\text{for } \alpha_t \quad &= \quad \frac{V_t}{V_t-H_t}\zeta
\end{aligned}
$$

- Certainty-equivalents are second-order equivalent
- Identical in continuous time
- I now take $\alpha_t=\frac{V_t}{V_t-H_t}\zeta$

$$V_t = \left\{ (1-\beta) C_t^{1-\rho} + \beta \left(E_t V_{t+1}^{1-\alpha_t} \right)^{\frac{1-\rho}{1-\alpha_t}} \right\}^{\frac{1}{1-\rho}}$$

- Analytically tractable
- Computationally simple
- Representative agent exists

$$V_t = \left\{ (1-\beta) C_t^{1-\rho} + \beta E_t \left[V_{t+1}^{1-\alpha_t} \right]^{\frac{1-\rho}{1-\alpha_t}} \right\}^{\frac{1}{1-\rho}}$$

$$\alpha_{t+1} = (1-\phi)\bar{\alpha} + \phi\alpha_t - \lambda \log \left(V_{t+1}^A / E_t V_{t+1}^A \right)$$

- Directly specify process for risk aversion
  - Equivalent to choosing process for H_t: average of past levels of V_t^A
  - V_t^A: aggregate value (external to the household)
  - Good news reduces risk aversion
    - News is forward-looking
- Risk aversion is not free/latent
  - Just need to measure V_t^A

The pricing kernel

- Pricing kernel (stochastic discount factor) satisfies for any asset return

$$1 = E_t \left[M_{t+1} R_{t+1} \right]$$

with

$$\begin{aligned} M_{t+1} &= \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{V_{t+1}}{E_t \left[V_{t+1}^{1-\alpha_t} \right]^{\frac{1}{1-\alpha_t}}} \right)^{\rho - \alpha_t} \\ &= \beta^{\frac{1-\alpha_t}{1-\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho \frac{1-\alpha_t}{1-\rho}} R_{w,t+1}^{\frac{\rho-\alpha_t}{1-\rho}} \end{aligned}$$

- $R_{w,t+1}$ return on a consumption claim
- Hansen–Jagannathan bound: $SD \left(M_{t+1} \right) / E \left[M_{t+1} \right]$ is maximum possible Sharpe ratio
  - Uncertainty over α_{t+1} raises volatility of $R_{w,t+1}$
  - Shocks to the price of risk are priced

Outline

1. Preference specification
2. **Production economy, solution**
    - **Match real and financial moments**
    - **Find substantial stock return predictability**
3. Estimate of risk aversion
4. EIS estimates
5. Extensions

Production and resource constraint

- Ouput is produced only with capital

$$Y_t = A_t^{1-\gamma} K_t^{\gamma}$$

- Technology:
  - Benchmark

$$\log A_t = \log A_{t-1} + \mu + \sigma_a \varepsilon_{a,t}$$

  - Dual-shock

$$\log A_t = \underbrace{\log \bar{A}_t}_{\text{Random Walk}} + \underbrace{\log X_t}_{\text{AR(1)}}$$

  - X_t captures temporary shocks to output
- Budget constraint

$$C_t + K_{t+1} = Y_t + (1-\delta) K_t$$

Parameters

- $\gamma = 0.33$ – capital's share of income
- $\delta = 0.08$ (annual)
- $\sigma_a = 0.0088$ – Permanent productivity shocks calibrated to match long-run variance of consumption growth
  - 80% lower than Kaltenbrunner and Lochstoer (2010) and Bansal and Yaron (2004)
- $\rho = 1/1.5$ – matches some micro studies; in general want $EIS = (1/\rho) > 1$
- $E[\alpha_t] = 14$; $SD[\alpha_t] = 6.2$ – calibrated to generate a large and volatile Sharpe ratio
- $\phi = 0.94$ – (persistence of RRA) empirical persistence of P/E; Half life=11 quarters

Solution

- Model is solved with projection
  - Approximate utility (V_t) and consumption functions with cubic polynomials
  - Solve two functions: V recursion and capital Euler equation
- Magnitude of errors:
  - Capital mispriced by $<$0.01bp
  - V recursion error $<10^{-9}$

Simulated macro moments

	Data	EZ-CRRA	EZ-habit	Dual-shock
Long-run SD	0.88	0.88	0.88	0.88
std(ΔC_t) (%)	0.46	0.28	0.47	0.56
std(ΔY_t) (%)	0.99	0.59	0.59	1.03
std(ΔI_t) (%)	2.65	1.11	0.83	2.37

- EZ-CRRA identical to EZ-habit but with constant relative risk aversion
- There are long-run risks in consumption (Kaltenbrunner and Lochstoer)

Simulated macro moments

	Data	EZ-habit	CC	Yang
Long-run SD	0.88	0.88	0.88	0.88
std(ΔC_t) (%)	0.46	0.47	**0.15**	**0.18**
std(ΔY_t) (%)	0.99	0.59	0.65	0.65
std(ΔI_t) (%)	2.65	0.83	2.17	3.91

- Habits induce smooth consumption growth
- Yang:

$$V_t = \left\{ (1-\beta)\left(C_t - H_t\right)^{1-\rho} + \beta \left[E_t V_{t+1}^{1-\alpha} \right]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}$$

Simulated financial moments

	Data	CRRA	EZ-habit	Dual-shock
Mean HJ bound	0.32	0.22	0.32	0.32
Mean exc. equity return	6.78	1.04	4.17	4.15
std(exc. equity return)	21.19	4.71	13.30	12.98
Mean risk-free rate	0.91	2.20	2.04	1.94
std(risk-free rate)	1.16	0.21	0.25	0.26

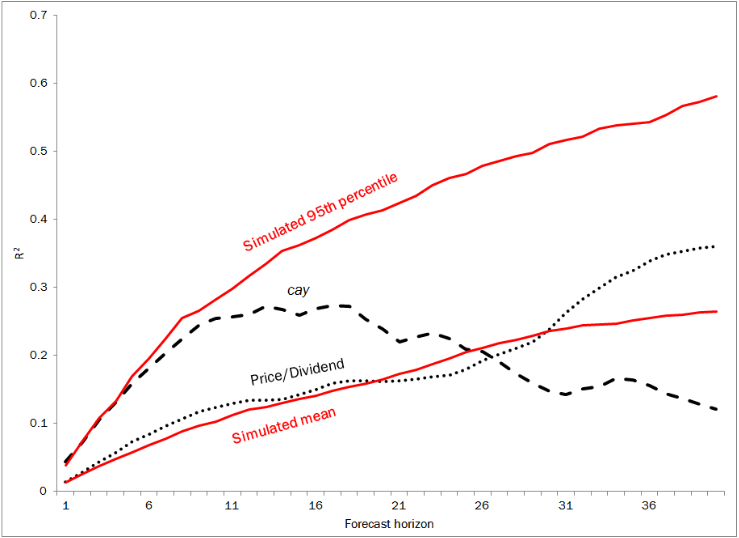
- Habits raise HJ bound by half, quadruple equity return
- Interest rates low and stable

Simulated financial moments

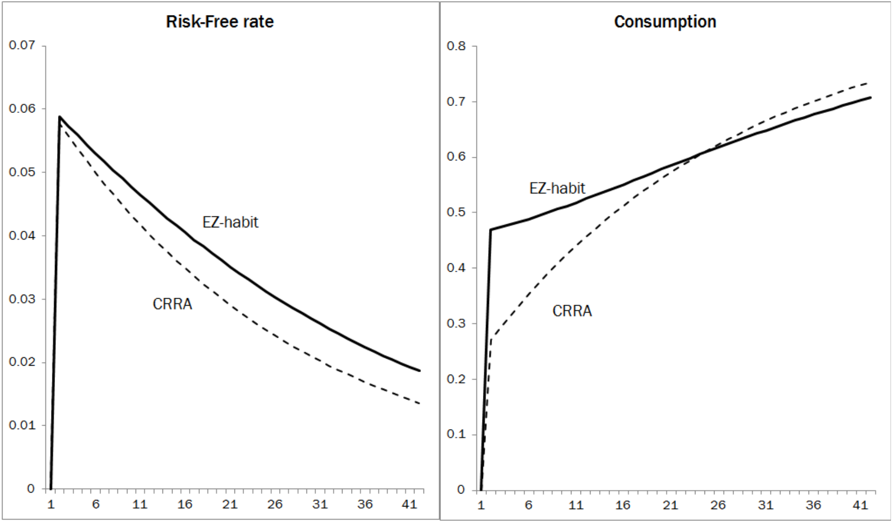
	Data	EZ-habit	CC	Yang	Jermann
Mean HJ bound	0.32	0.32	**0.071**	**0.05**	0.34
Mean exc. equity return	6.78	4.17	0.13	0.14	1.48
std(exc. equity return)	21.19	13.30	3.79	2.81	13.18
Mean risk-free rate	0.91	2.05	4.98	15.81	0.08
std(risk-free rate)	1.16	0.25	0.77	1.18	**10.44**

- Standard deviation of HJ bound for CC and Yang is near zero
- Jermann (1998): habits, adjustment costs in investment

The degree of predictability



Impulse response functions (percentage points)



Outline

1. Preference specification
2. Production economy, solution
3. **Estimate of risk aversion**
    - **Empirical test of the model; direct measure of risk aversion**
    - **Separate from production model**
4. EIS estimates
5. Extensions

Measuring risk aversion

- Define the *Wealth Portfolio* as the asset that pays C_t as its dividend
- For Epstein-Zin preferences,

$$W_t = V_t^{1-\rho} C_t^{\rho} / (1-\beta)$$

intuitively, $W = V / MU_C$

$$V_t \propto W_t \left(W_t / C_t \right)^{\frac{\rho}{1-\rho}}$$

$\Longrightarrow$ With value, we can calculate risk aversion

$$\alpha_{t+1} = (1-\phi)\,\bar{\alpha} + \phi\alpha_t - \lambda \log\left(V_{t+1}^A / E_t V_{t+1}^A \right)$$

- Check for whether the assumed process for α_t fits returns
- *Requires no knowledge of consumption dynamics or production*

Measuring risk aversion

Plugging in the formula for V_t^A:

$$\begin{aligned} \alpha_{t+1} &= (1-\phi)\,\bar{\alpha} + \phi\alpha_t - \lambda \log\left(V_{t+1}^A / E_t V_{t+1}^A \right) \\ &= (1-\phi)\,\bar{\alpha} + \phi\alpha_t - \lambda \begin{pmatrix} \Delta E_{t+1} \log W_{t+1} \\ +\frac{\rho}{1-\rho}\Delta E_{t+1} \log\left(W_{t+1} / C_{t+1} \right) \end{pmatrix} \end{aligned}$$

Calibrate ρ and ϕ; ignore $\bar{\alpha}$ and λ

Estimates risk aversion up to a normalization

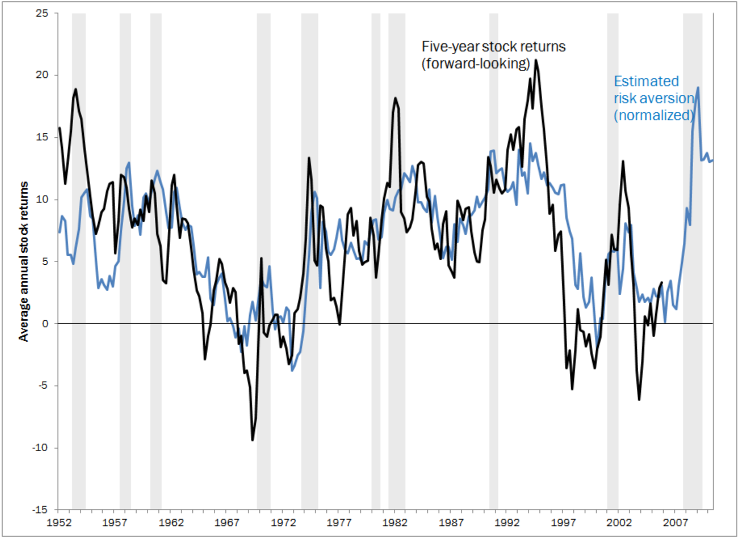
Data

- Measure wealth as in Lettau and Ludvigson (cay, 2001)
  - Financial wealth from the Flow of Funds
  - Proxy for human wealth with labor income (BEA)

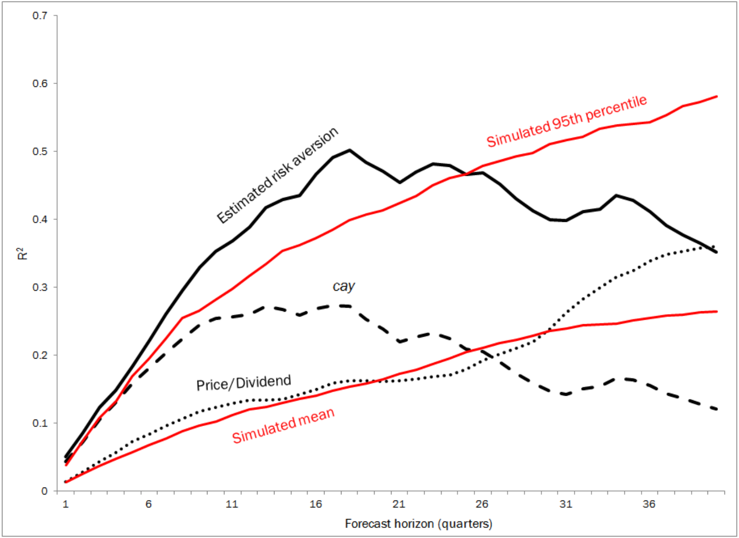
$$\hat{w}_t = \kappa a_t + (1 - \kappa) y_t$$

  - κ estimated from a VECM in c_t, a_t and y_t; $\kappa \approx 1/3$
- Consumption is from the BEA
- CRSP value-weighted excess return
- All data is post-war quarterly – 1952–2010 (constraint is Flow of Funds data)

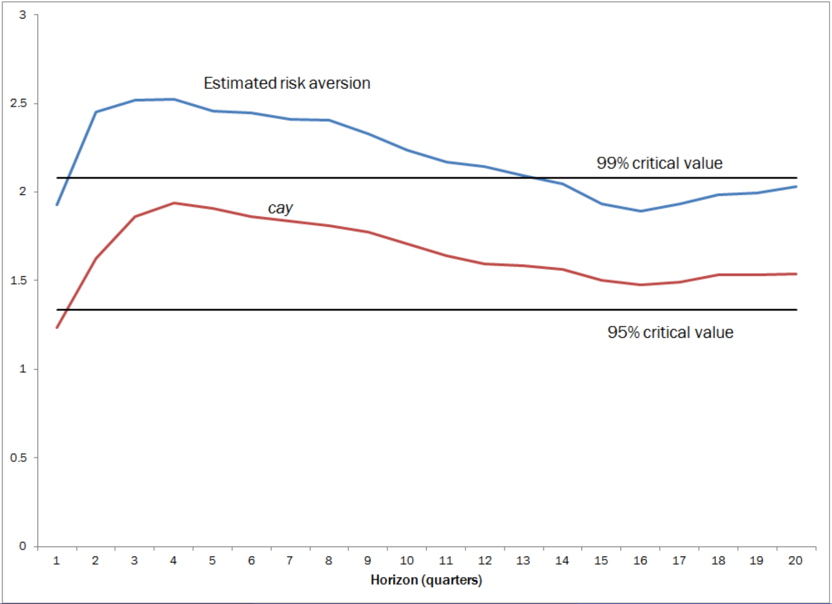
Risk aversion and 5-year returns



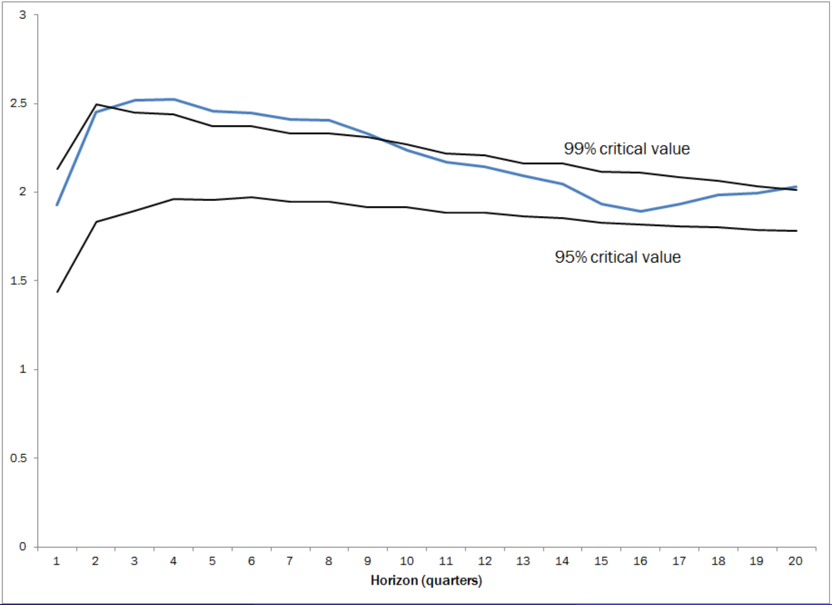
Long-horizon R2s (univariate regressions)



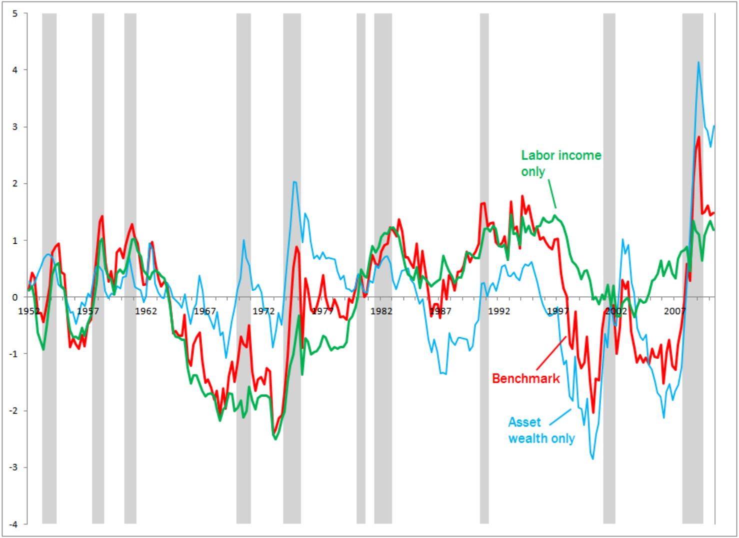
Out-of-sample test statistics



Boostrapped critical values



Are the results driven by asset wealth?



Risk aversion measured from technology growth

- In production model, permanent innovations to technology drive α_t
- Beveridge–Nelson decomposition

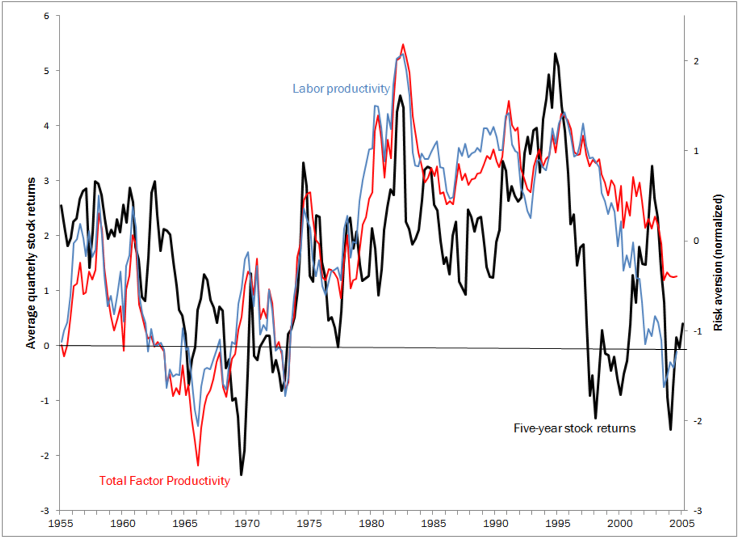
$$\begin{aligned} prod_t &= prod_t^* + u_t \\ E_t prod_{t+1}^* &= prod_t^* + g \end{aligned}$$

  - Estimate ARMA model for *prod* to extract *prod**
- Then

$$\hat{\alpha}_t = (1 - \phi)\,\bar{\alpha} + \phi \hat{\alpha}_{t-1} - \lambda \left(\Delta prod_t^* - g \right)$$

- Use labor productivity or TFP growth
- Not implied by disaster models

Risk aversion measured from technology growth



Outline

1. Preference specification
2. Production economy, solution
3. Estimate of risk aversion
4. **EIS estimates**
    - **Euler equation estimation is biased**
5. Extensions

EIS regressions

- Standard EIS regression

$$E_t \Delta c_{t+1} = \rho^{-1} r_{f,t+1} + \alpha_t PS$$

with constant α_t, identifies EIS=ρ^{-1}

$PS =$ precautionary saving effect

- Suppose α_t not constant,

$$\hat{\rho}^{-1} = \rho^{-1}\left(1 + \frac{cov\left(r_{f,t+1}, \alpha_t PS\right)}{var\left(r_{f,t+1}\right)}\right)$$

Bias: $r_{f,t+1}$ is negatively correlated with α_t through time-varying precautionary saving

- Measure $r_{f,t+1}$ as the three-month nominal rate minus an inflation forecast

EIS regressions

Hall (1988), Campbell and Mankiw (1989) regression:

$\Delta c_{t+1} = b_0 + b_1 r_{f,t+1} + \varepsilon_{t+1}$

	Data	CRRA	EZ-habit
Median	0.13	1.16	0.03
[2.5, 97.5]	N/A	[0.03, 1.79]	[-1.98, 1.02]

Model-consistent regression:

$\Delta c_{t+1} = b_0 + b_1 r_{f,t+1} + b_2 \alpha_t + \varepsilon_{t+1}$

	Data	CRRA	EZ-habit
Median	0.08	N/A	0.07
[2.5, 97.5]	N/A	N/A	[-3.08, 3.11]

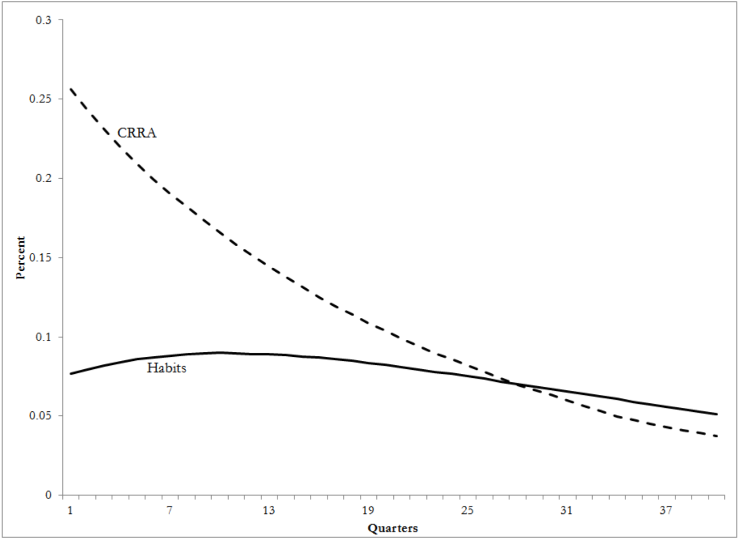
Outline

1. Evidence on return predictability
2. Preference specification
3. Producution economy, solution
4. Estimate of risk aversion; EIS estimates
5. **Extensions**

Labor supply

- Habits also interfere with labor supply (Boldrin, Christiano, and Fisher, 2001)
  - Volatility in marginal utility of consumption makes labor very volatile
  - Reduces risk premia because households self-insure
- Not a problem here:
  - Risk aversion does not affect marginal utility
  - Self-insurance does not affect risk premia

Labor supply IRF



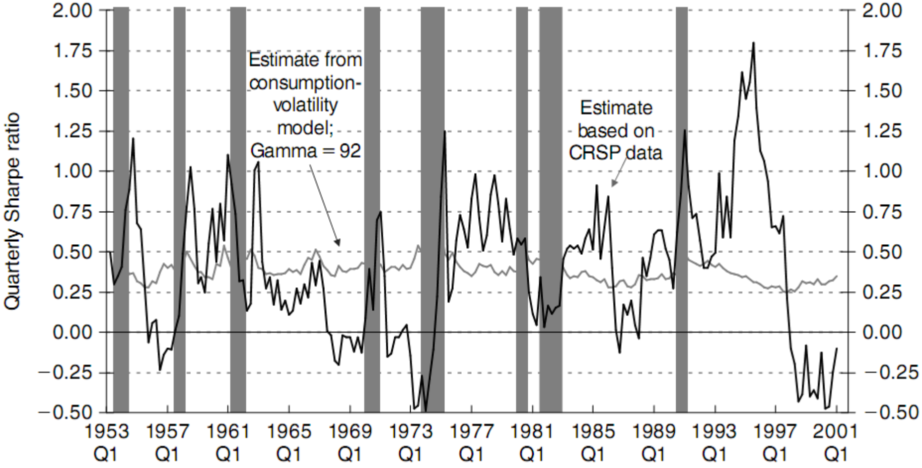
Extensions

- Model is highly tractable
  - Generates an essentially affine SDF
  - Merges standard frameworks in macro and asset pricing
- Simple extensions:
  - Uncovered interest parity puzzle
  - General-equilibrium bond pricing
  - Financial frictions
- Companion paper does asset pricing in a rich macro model
  - Fully dynamic estimation (Kalman filter)

Conclusion

- I build a new preference specification
  - Epstein–Zin + Habits in continuation value
- Matches standard macro moments, comovement
- Implies realistic return predictability
- Gives strong empirical return forecasts
- Explains why the EIS is estimated to be small
- Simple, tractable framework for modeling time-varying risk prices

Lettau and Ludvigson's estimate of the Sharpe ratio



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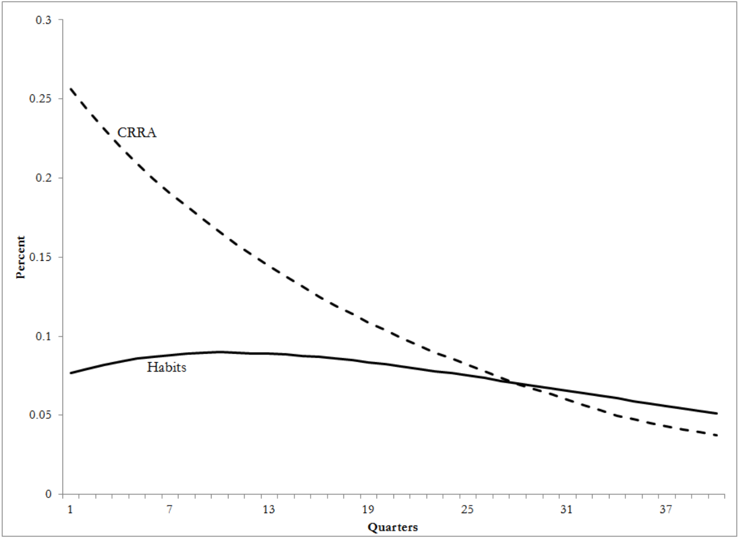
Labor supply IRF



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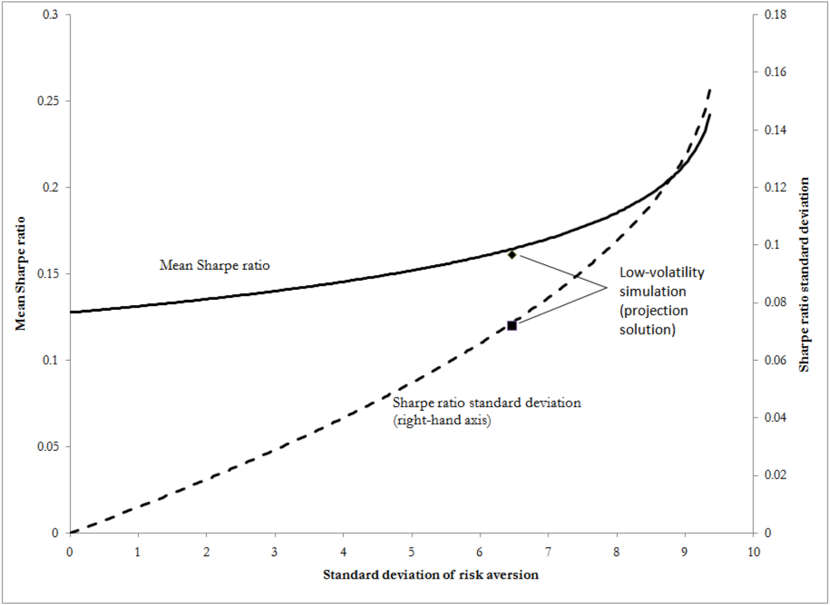
0.3
0.25
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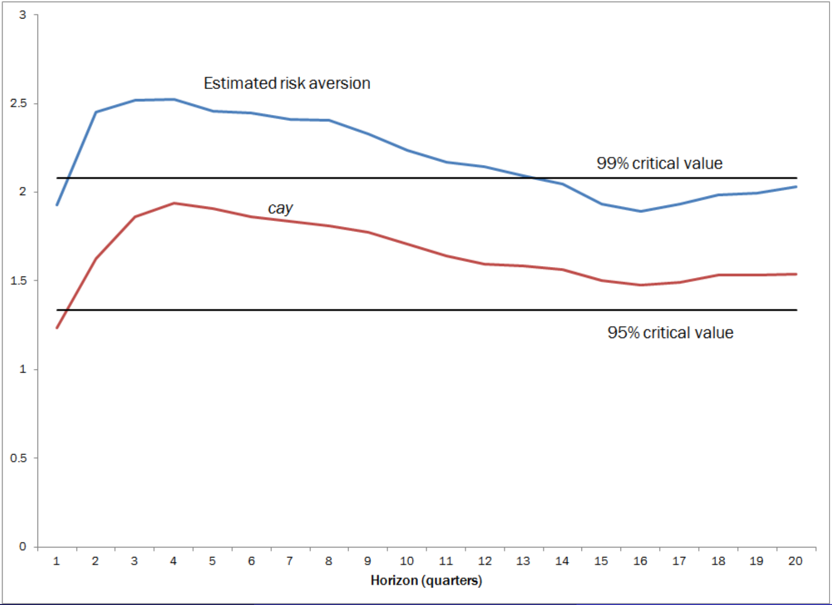
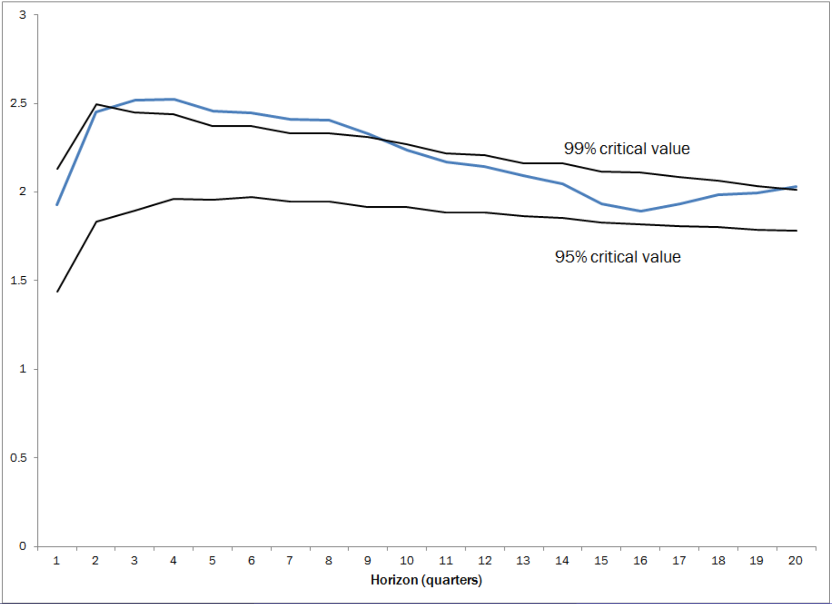
Mean Sharpe ratio
Sharpe ratio standard deviation
Standard deviation of risk aversion
Mean Sharpe ratio
Low-volatility simulation (projection solution)
Sharpe ratio standard deviation (right-hand axis)

Out-of-sample test statistics



Boostrapped critical values



Encompassing tests