Sharing in Ynot

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January 15, 2010
Outline

1 Verification
   - Ynot

2 Lists in Ynot

3 Sharing: Iterators

4 Aliasing: B+ Trees

5 The Burden of Proof
Outline

1 Verification
   • Ynot

2 Lists in Ynot

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4 Aliasing: B+ Trees

5 The Burden of Proof
Gaining Assurance

Observation
If there’s one thing that we’ve learned in the past 20 years it’s that all software has bugs.

- Tried and are trying a lot of approaches to mitigate this problem:
  - (Unit) Testing
  - Bug Finding Tools
  - Static Type Systems
  - Model Checking
  - Theorem Proving
Gaining Assurance

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  - Model Checking
  - Theorem Proving
The Burden of Proofs

Several projects have worked on verification.
- Jahob - Verification in Java
- Spec# - Verification in C#
- seL4 - Kernel verification in Agda

Standard approach to verification:
1. Write specifications.
2. Write all of the code.
3. Give specifications and code to VC Generator.
4. Modify code/add annotations until and repeat until verification succeeds.
Most type systems don’t express side-effects explicitly.

(* swap : 'a ref -> 'a ref -> unit *)
Most type systems don’t express side-effects explicitly.

```ml
(* swap : 'a ref -> 'a ref -> unit *)
let swap a b =
  let t = !a in
  a := !b ;
  b := t
```

- Simplifies coding.
- But the types don’t tell us whether a function is really a function!
Explicit IO: Haskell

- Haskell makes side-effects explicit using monads.

```haskell
swap :: MVar a -> MVar a -> IO ()
```
Explicit IO: Haskell

- Haskell makes side-effects explicit using monads.

\[
\text{swap} :: \text{MVar } a \to \text{MVar } a \to \text{IO } ()
\]

\[
\text{swap } p1 \ p2 =
\begin{align*}
\text{do } \{ & \ t1 \leftarrow \text{takeMVar } p1 \\
& \ t2 \leftarrow \text{takeMVar } p2 \\
& \ \text{putMVar } p1 \ t2 \\
& \ \text{putMVar } p2 \ t1
\}
\end{align*}
\]

- Can now determine if a function doesn't have side effects.
- Only looking at the type, we know more, but not enough.
Specifications: Ynot

- Hoare logic-based specifications using dependent types.
- Index the IO monad by pre- and post-conditions.
  - Allows us to precisely specify the effects of a computation.

**Definition** swap : forall (p1 p2 : ptr) (v1 v2 : [nat]),

Cmd (v1 ~~ v2 ~~ p1 ~~> v1 * p2 ~~> v2)

(fun _ : unit => v1 ~~ v2 ~~ p1 ~~> v2 * p2 ~~> v1).

DEMO
Specifications: Ynot

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Definition swap : forall (p1 p2 : ptr) (v1 v2 : [nat]),
  Cmd (v1 ~~ v2 ~~ p1 ~~> v1 * p2 ~~> v2)
  (fun _ : unit => v1 ~~ v2 ~~ p1 ~~> v2 * p2 ~~> v2).
refine (fun p1 p2 v1 v2 =>
  t1 <- ! p1 ;
  t2 <- ! p2 ;
  p1 ::= t2 ;;
  {{ p2 ::= t1 }});
sep fail auto. (** Proof **) Qed.
Overview

1. Logic
   - Shallow embedding of separation logic.
   - Computational irrelevance.

2. Monad
   - Cmd monad indexed by pre- and post-conditions.

3. Tactics
   - Ltac automation for separation logic.
Separation Logic

(** Predicates over heaps **)  
Definition heap := ptr -> option Dyn.  
Definition hprop := heap -> Prop.
Separation Logic

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Definition emp : hprop := fun h => forall p, h p = None.
Separation Logic

(** Predicates over heaps **) 
Definition heap := ptr -> option Dyn. 
Definition hprop := heap -> Prop. 

Definition emp : hprop := fun h => for all p, h p = None. 

Definition cell p v : hprop := fun h => 
  for all p', if p = p' then h p = Some v 
  else h p = None.
(** Predicates over heaps **)  

**Definition** heap := ptr \rightarrow \text{option Dyn}.

**Definition** hprop := heap \rightarrow \text{Prop}.

**Definition** emp : hprop := fun h \Rightarrow \forall p, h p = \text{None}.

**Definition** cell p v : hprop := fun h \Rightarrow
\begin{align*}
&\forall p', \text{if } p = p' \text{ then } h p = \text{Some v} \\
&\quad \text{else } h p = \text{None}.
\end{align*}

**Definition** hprop_sep (P Q : hprop) : hprop :=
fun h \Rightarrow \exists h1 h2, h \Rightarrow h1 \ast h2 \land P h1 \land Q h2.
Axiom Cmd : \( \forall \) (pre : hprop) \( \{A\}\) (post : \( A \rightarrow hprop\)), Set.
Axiom Cmd : forall \( \text{pre} : \text{hprop} \) \{A\} (\( \text{post} : A \rightarrow \text{hprop} \)), \text{Set}.

Axiom CmdBind : forall \( \text{pre1} T1 \) (\( \text{post1} : T1 \rightarrow \text{hprop} \))
\( \text{pre2} T2 \) (\( \text{post2} : T2 \rightarrow \text{hprop} \))
(\( \text{st1} : \text{Cmd} \text{pre1} \text{post1} \))
(\( \_ : \text{forall} \ v, \text{post1} \ v \Rightarrow \text{pre2} \ v \))
(\( \text{st2} : \text{forall} \ v : T1, \text{Cmd} (\text{pre2} \ v \text{post2}) \))
: \text{Cmd} \text{pre1} \text{post2}.
Ynot Library : Command Monad

Axiom Cmd : forall (pre : hprop) {A} (post : A -> hprop), Set.

Axiom CmdBind : forall pre1 T1 (post1 : T1 -> hprop) pre2 T2 (post2 : T2 -> hprop)
(st1 : Cmd pre1 post1)
(_ : forall v, post1 v ==> pre2 v)
(st2 : forall v : T1, Cmd (pre2 v) post2)
: Cmd pre1 post2.

Axiom CmdRead : forall (T : Set) (p : ptr) (P : T -> hprop),
Cmd (Exists v :@ T, p ~> v * P v)
(fun v => p ~> v * P v).

(** ... and more ... **)
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2. Lists in Ynot

3. Sharing: Iterators

4. Aliasing: B+ Trees

5. The Burden of Proof
C-style Linked Lists

- Linked lists in ML.

```ocaml
module type LLIST =
struct
  type 'a t
  val new : unit -> 'a t
  (** ... **) 
  val sub : 'a t -> int -> 'a option
end
```

- A type (t) and functions on it (new, sub).
C-style Linked Lists

- Linked lists in ML.

```ocaml
module type LLIST =
struct
  type 'a t
  val new : unit -> 'a t
  (** ... **)
  val sub : 'a t -> int -> 'a option
end
```

- A type (t) and functions on it (new, sub).
- To reason about correctness, we need specifications.
  1. Relate the type t to a computationally irrelevant model.
  2. Provide a predicate that describes the heap in terms of model.
  3. Provide specifications as stronger types for the functions.
Representation Predicate

Describe the heap computationally using a functional model.

```
F i x p o i n t llseg (pStart pEnd : optr) (ls : list T) : hprop :=
  m a t c h ls with
  | nil => [ pStart = pEnd ]
  | a :: b =>
    m a t c h pStart with
    | None => [ False ]
    | Some p =>
      E x i s t s nx :@ option ptr, p ~~ > mkNode a nx * llseg nx pEnd b
  end end.
```

```
D e f i n i t i o n tlst := ptr.
D e f i n i t i o n llist (h : tlst) (m : list T) : hprop :=
  E x i s t s st :@ option ptr, h ~~ > st * llseg st None m.
```
Representation Predicate

pStart ................................. pEnd

- Describe the heap computationally using a functional model.

\[
\text{Fixpoint } \text{llseg} \ (p\text{Start} \ p\text{End} : \text{o}ptr) \ (ls : \text{list} \ T) : \text{hprop} := \\
\text{match } ls \ \text{with} \\
| \ \text{nil} \quad \Rightarrow \ [p\text{Start} = p\text{End}]
\]
Describe the heap computationally using a functional model.

Fixpoint llseg (pStart pEnd : optr) (ls : list T) : hprop :=
  match ls with
  | nil   => [pStart = pEnd]
  | a :: b => match pStart with
      | None => [False]
Describe the heap computationally using a functional model.

Record \texttt{llNode} := \texttt{mkNode} \{ \texttt{val} : \texttt{T} ; \texttt{next} : \texttt{optr} \}.

Fixpoint \texttt{llseg} (\texttt{pStart} \ \texttt{pEnd} : \texttt{optr}) (\texttt{ls} : \texttt{list} \ \texttt{T}) : \texttt{hprop} :=
match \texttt{ls} with
| \texttt{nil} => [\texttt{pStart} = \texttt{pEnd}]
| \texttt{a} :: \texttt{b} => match \texttt{pStart} with
  | \texttt{None} => [False]
  | \texttt{Some} \texttt{p} => Exists \texttt{nx} @ option ptr,
  \texttt{p} ~> \texttt{mkNode} \texttt{a} \texttt{nx} * \texttt{llseg} \texttt{nx} \texttt{pEnd} \texttt{b}
end end.
Describe the heap computationally using a functional model.

Record llNode := mkNode { val : T ; next : optr }.

Fixpoint llseg (pStart pEnd : optr) (ls : list T) : hprop :=
match ls with
| nil => [pStart = pEnd]
| a :: b => match pStart with
  | None => [False]
  | Some p => Exists nx :@ option ptr, p ~~> mkNode a nx * llseg nx pEnd b
end end.

Definition tlst := ptr.
Definition llist (h : tlst) (m : list T) : hprop :=
Exists st :@ option ptr, h ~~> st * llseg st None m.
Definition sub : forall (t : tlst) (i : nat) (m : [list T]),
  Cmd (m ~~ llist t m)
   (fun res : option T =>
      m ~~ llist t m * [res = nth_error m i]).
Linked Lists: sub

Definition sub : forall (t : tlst) (i : nat) (m : [list T]),
    Cmd (m ~ llist t m)
    (fun res : option T =>
        m ~ llist t m * [res = nth_error m i]).
refine (fun t i m =>
    hd <- ! t ;
    {{ Fix3 (fun hd j m => m ~ llseg hd None m)
        (fun hd j m (r : option T) =>
            m ~ llseg hd None m * [r = nth_error m j])
        (fun self hd j m =>
            IfNull hd Then {{ Return None }}
            Else
                nde <- ! hd ;
                IfZero j Then
                    {{ Return (Some (val nde)) }}
                Else
                    {{ self (next nde) j (m ~ tail m) <$> _ }}
                ) hd i m <$> _ }});
    try clear self; sep’s tac.
Qed.
Linked Lists: \texttt{sub}

Definition \texttt{sub : forall (t : tlst) (i : nat) (m : [list T]), Cmd (m \texttt{~~} llist t m)}
\begin{verbatim}
  (fun res : option T =>
    m \texttt{~~} llist t m * [res = nth_error m i]).
\end{verbatim}
refine (fun t i m =>
  hd <- ! t ;
  {{ Fix3 (fun hd j m => m \texttt{~~} llseg hd None m)
  (fun hd j m (r : option T) =>
    m \texttt{~~} llseg hd None m * [r = nth_error m j])
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      nde <- ! hd ;
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        {{ Return (Some (val nde)) }}
      Else
        {{ self (next nde) j (m \texttt{~~} tail m) <@> _ }}
    ) hd i m <@> _ }});
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Qed.
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try clear self; sep’s tac.
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Linked Lists: sub

**Definition** sub : forall (t : tlst) (i : nat) (m : [list T]),

Cmd (m ~ llist t m)

(fun res : option T =>
  m ~ llist t m * [res = nth_error m i]).

refine (fun t i m =>
  hd <- ! t ;

{{ Fix3 (fun hd j m => m ~ llseg hd None m)
  (fun hd j m (r : option T) =>
    m ~ llseg hd None m * [r = nth_error m j])
  (fun self hd j m =>
    IfNull hd Then {{ Return None }}
    Else
      nde <- ! hd ;
      IfZero j Then
        {{ Return (Some (val nde)) }}
      Else
        {{ self (next nde) j (m ~ tail m) <@> _ }}
    )}})

try clear self; sep’s tac.

Qed.
Linked Lists: mfold_left

- Can even write higher-order computations while maintaining abstraction.

Definition mfold_left : forall {U : Type} (t : tlst) (I : list T -> U -> hprop) (a : U) (m : [list T]),
  Cmd (m ~ I m a)
  (fun a : U => m ~ I (m ++ c :: nil) a),
Cmd (m ~ llist t m * I nil a)
  (fun r : U => m ~ llist t m * I m r).

- I is the invariant.
- a is the initial accumulator.
- cmd is the folded computation.
Outline

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2. Lists in Ynot

3. **Sharing: Iterators**

4. Aliasing: B+ Trees

5. The Burden of Proof
Adding Iterators

- Iterators and collections go hand-in-hand.

Class ListIterable (h : Set) (T : Type) : Type := {
  rep : h -> list T -> nat -> hprop ;
}.

h is the type of the iterator handle.
T is the type of values being iterated over.
Representation predicate (rep) and next command.
Adding Iterators

- Iterators and collections go hand-in-hand.

Class ListIterable (h : Set) (T : Type) : Type := {
  rep : h -> list T -> nat -> hprop ;
  next : forall (t : h) (m : [list T]) (idx : [nat]),
      Cmd (m ~~ idx ~~ rep t m idx)
   (fun res : option T => m ~~ idx ~~
      rep t m (nextIndex idx (length m)) * 
      [res = nth_error m idx])
}.

- h is the type of the iterator handle.
- T is the type of values being iterated over.
- Representation predicate (rep) and next command.
Definition titr := ptr.

(** Representation predicate **)  
Definition liter (t : titr) (ls : list T) (idx : nat) :=
  hprop :=
  Exists st :@ optr, Exists cur :@ optr,
  t ~~> (cur, st) *
A Naïve Iterator

Definition titr := ptr.

(** Representation predicate **)  
Definition liter (t : titr) (ls : list T) (idx : nat) :
    hprop :=  
    Exists st : optr, Exists cur : optr,
    t ~> (cur, st) *  
    llseg st cur (firstn idx ls) *
A Naïve Iterator

Definition titr := ptr.

(** Representation predicate **)  
Definition liter (t : titr) (ls : list T) (idx : nat) : hprop :=

.Exists st : @ optr, Exists cur : @ optr,

\( t \leadsto (cur, st) \) *

llseg st cur (firstn idx ls) *

llseg cur None (skipn idx ls).
The Sharing Problem

- Requires access to the same memory as the underlying list.
  - Creating an iterator *consumes* the underlying list.
  - Can’t have multiple iterators.

```
  h
  \arrow{\rightarrow}
  \begin{array}{c}
  'A'
  \end{array}
  \rightarrow
  \begin{array}{c}
  'B'
  \end{array}
  \rightarrow
  \begin{array}{c}
  'C'
  \end{array}
  \rightarrow
  \begin{array}{c}
  \text{List}
  \end{array}
```
The Sharing Problem

- Requires access to the same memory as the underlying list.
  - Creating an iterator *consumes* the underlying list.
  - Can’t have multiple iterators.

Diagram:

```
  i
  v
  'A' -- 'B' -- 'C'
  \---/     \   \
    \       \  
      \      \ 
        \    /  
          \  /   
            \ /    
              \    
              \   
                \  
                  \ 
                    \ 
                      \ 
```

Iterator

Gregory Malecha (Harvard University SEAS)
The Sharing Problem

- Requires access to the same memory as the underlying list.
  - Creating an iterator *consumes* the underlying list.
  - Can’t have multiple iterators.
The Sharing Problem: Specifications

- Computations on iterators can’t be called with the same underlying list.

**Definition** `zip : forall (i1 i2 : titr)
  (l1 : [list T]) (l2 : [list U]),
  Cmd (l1 ~~ l2 ~~ liter i1 l1 0 * liter i2 l2 0 *
    [length l1 = length l2])
  (fun res : tlst => l1 ~~ l2 ~~
    liter i1 l1 (length l1) * liter i2 l2 (length l2) *
    llist res (zip l1 l2))`
A Real Sharing Problem

Who “owns” the list turns out to be a real problem.
A Real Sharing Problem

- Who “owns” the list turns out to be a real problem.
A Real Sharing Problem

Who “owns” the list turns out to be a real problem.

Source of Java’s ConcurrentModificationException.
A Real Sharing Problem

- Who “owns” the list turns out to be a real problem.

- Doesn’t satisfy frame property!
  - Source of Java’s ConcurrentModificationException.
Parameterize points-to by a fractional ownership.

- $p \sim[q] \rightarrow v$, $q$ is the fraction.

Ownership determines your capabilities:
- Full permissions allows everything: read, write, free.
- Partial permissions only allows reading.
- Permissions can be split and joined.

---

1Ynot implementation by Avi Shinnar.
A Fractional Iterator

- Describe the iterator as owning a fraction of the whole list.

(** Representation predicate **)

**Definition** liter (owner : tlst) (q:Fp) (t : titr) (ls : list T) (idx : nat) : hprop :=

- Exists st @(optr), Exists cur @(optr),
- t ~> (cur, st) *
- llseg st cur (firstn idx ls) q *
- llseg cur None (skipn idx ls) q.

- q is the fraction of the list that is owned.
- Allows multiple iterators over the same list.
  - As long as the fractions are compatible.
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Data structures with aliasing are more difficult to describe.
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B+ Trees

- B+ trees are \( n \)-ary trees where the leaves are connected by a linked list.
  - Support fast lookup and in-order iteration.
  - Commonly used for database indices. (Malecha ’10)
- Previous formalizations exist, but neither is mechanically verified:
  - Classical conjunction, \((\text{list} \times \text{any}) \land \text{tree}\). (Bornat ’04)
  - B+ tree language. (Sexton ’08)
- Both of these approaches seemed difficult to automate.
Difficulties of the Invariant

Several difficulties describing this:
- Have to encode pointer aliasing explicitly.
- Many different B+ trees can describe the same finite map.
- Enforce the tree balancedness.
- Enforce the ordering of keys.
- Invariants on the size of branches and leaves.
Difficulties of the Invariant

- Several difficulties describing this:
  - Have to encode pointer aliasing explicitly.
  - Many different B+ trees can describe the same finite map.
  - Enforce the tree balancedness.
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  - Invariants on the size of branches and leaves.
Representation Invariant

- Existentially quantify an irrelevant model ($tr$) of the tree which contains the pointers.
  - Avoids existentials in the representation invariant, simplifies automation.
  - Makes the heap predicate ($repTree$) very computational.

Definition $rep (p : BptMap) (m : Model) : hprop :=$

$$\exists pRoot :@ ptr, \exists h :@ nat, \exists tr :@ ptree h, p \rightsquigarrow (pRoot, \exists T (fun h:nat => [ptree h]) h [tr]) *$$

$$repTree pRoot None tr *$$
Representation Invariant

- Existentially quantify an irrelevant model \((tr)\) of the tree which contains the pointers.
  - Avoids existentials in the representation invariant, simplifies automation.
  - Makes the heap predicate \((\text{repTree})\) very computational.
  - Connect the logical model \((m)\) to the physical model \((tr)\).

\[
\text{Definition } \text{rep} \ (p : \text{BptMap}) \ (m : \text{Model}) : \text{hprop} := \\
\exists p\text{Root} :@ \text{ptr},\ \exists h :@ \text{nat},\ \exists tr :@ \text{ptree} h,\ \\
p \rightsquigarrow (p\text{Root}, \text{existT} (\text{fun} h:\text{nat} => [\text{ptree} h]) h [tr]) \ast \\
\text{repTree} p\text{Root} \text{None} tr \ast \\
[\text{eqlistA entry_eq} m (\text{as_map} tr)] \ast
\]
Representation Invariant

- Existentially quantify an irrelevant model (tr) of the tree which contains the pointers.
  - Avoids existentials in the representation invariant, simplifies automation.
  - Makes the heap predicate (repTree) very computational.
  - Connect the logical model (m) to the physical model (tr).
  - Consolidate pure facts about the model in inv.

**Definition** \(\text{rep} (p : \text{BptMap}) (m : \text{Model}) : \text{hprop} :=\)

\[
\exists \text{pRoot} : @ \text{ptr}, \exists \text{h} : @ \text{nat}, \exists \text{tr} : @ \text{ptree} \text{ h, p} \leadsto (\text{pRoot}, \text{existT} (\text{fun} \ h : \text{nat} \Rightarrow [\text{ptree} \text{ h}]) \text{ h [tr]}) \ast \\
\text{repTree} \text{ pRoot None tr} \ast \\
[\text{eqlistA entry_eq m (as_map tr)}] \ast \\
[\text{inv _ tr MinK MaxK}].
\]
1 Model, 2 Views

- Can switch between views by proving and applying a lemma:

\[
\text{Lemma } \text{repTree}_\text{repTrunkLeaves} : \text{forall } (h : \text{nat}) \ (p : \text{ptr}) \ (optr : \text{optr}) \ (m : \text{ptree } h), \\
\text{repTree } p \text{ optr } m \\
\iff \\
\text{repTrunk } p \text{ optr } m \star \\
\text{repLeaves } (\text{Some } (\text{firstPtr } m)) \ (\text{leaves } m) \text{ optr}.
\]
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Some Lessons

- Fractional permissions are necessary even for sequential code.
- Separation logic makes trees much easier than DAGs/graphs.
  - Can simplify things by reifying an irrelevant model.
  - Big win for automation.
- Higher-order ADT functions: fold
- Automation pays off when reasoning about separation logic.
- Higher-order abstraction simplifies specifications and proofs.
Other Projects & Outlook

Previous Projects

- Verified web application — trace-based I/O. (Wisnesky ’09)
- Verified relational database. (Malecha ’10)
Other Projects & Outlook

Previous Projects
- Verified web application — trace-based I/O. (Wisnesky ’09)
- Verified relational database. (Malecha ’10)

Future?
- Still a fair amount of work for a more realistic system.
  - Reasoning about concurrency.
    - Brookes ’07, Appel ’08, Nanevski ’09
  - Reasoning about failures.
  - Proofs can still be tedious & long.
    - Domain specific external provers.

http://ynot.cs.harvard.edu/