Math E-15 Graduate Seminar - Day One

Since the idea of limit is the foundation on which calculus is built, we need a solid understanding of what limits are.

Big Question. What do we really mean when we write $\lim_{x\to c} f(x) = L$?

Informal Definition. The statement $\lim_{x\to 3} f(x) = 5$ means that by picking a domain of x-values close enough (but not equal) to 3, we can guarantee that the value of $f(x)$ will be as close (but not necessarily equal) to 5 as we like. We're concerned with the function's behavior *near* $x = 3$, not *at* $x = 3$.

We will make this definition more formal in the weeks to come. For now, we look at some examples. In each case, by filling in the table of values and sketching the graph of the function, we will try to determine what the limit should be.

Example 1. $f(x) = \dfrac{x^2 - 1}{x - 1}$

x	0.9	0.99	1	1.01	1.1
$f(x)$					

Conclusion: $\lim_{x\to 1} f(x) =$

Example 2. $f(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & \text{if } x \neq 1 \\ 3, & \text{if } x = 1 \end{cases}$

x	0.9	0.99	1	1.01	1.1
$f(x)$					

Conclusion: $\lim_{x\to 1} f(x) =$

Example 3. $f(x) = \begin{cases} \dfrac{\sin(3x)}{x}, & \text{if } x \neq 0 \\ 3, & \text{if } x = 0 \end{cases}$

Note: for reasons that will become clear, we use radians in calculus.

x	-0.1	-0.01	0	0.01	0.1
$f(x)$					

Conclusion: $\lim_{x\to 0} f(x) =$

Example 4. $f(x) = \begin{cases} x, & \text{if } x < 1 \\ x+1, & \text{if } x \geq 1 \end{cases}$

x	0.9	0.99	1	1.01	1.1
$f(x)$					

Conclusion: $\lim_{x\to 1} f(x) =$

Example 5. $f(x) = \sin(1/x)$

x	-0.1	-0.01	0	0.01	0.1
$f(x)$					

Conclusion: $\lim_{x \to 0} f(x) =$

Example 6. $f(x) = \begin{cases} \dfrac{x^3 - 8}{x - 2}, & \text{if } x \neq 2 \\ 10, & \text{if } x = 2 \end{cases}$

x	1.9	1.99	2	2.01	2.1
$f(x)$					

Conclusion: $\lim_{x \to 2} f(x) =$

Example 7. $f(x) = \dfrac{1 - \cos(3x)}{x^2}$

x	-0.1	-0.01	0	0.01	0.1
$f(x)$					

Conclusion: $\lim_{x \to 0} f(x) =$

Next Week. We will refine our definition of the limit to remove mathematically ambiguous terms such as “close enough” and practice determining just how “close” we mean by “close enough.” This will cover pages 61 to the middle of page 63 (up to but not including Example 2 on that page) of the handout from today.