Naive Learning in Social Networks
and the Wisdom of Crowds

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  - Many interesting networks have poor learning; many also have good learning.
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- The vector of all beliefs is $\mathbf{b}(t) \in \mathbb{R}^n$.
- The initial beliefs $b_i(0)$ are independent random draws with mean $\theta$ and all lie in the same compact set $[-K, K]$. 
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For example:

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$$b_i(t + 1) = \sum_{j \in A} T_{ij} b_j(t)$$

where

$$\sum_{j \in A} T_{ij} = 1.$$
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\[ \Rightarrow \quad b(t) = T^t b(0). \]
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b(t + 1) = T b(t)
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\[\Rightarrow b(t) = T^t b(0).\]

Also, \( \sum_{j \in A} T_{ij} = 1 \Rightarrow \) each row of \( T \) sums to 1.
The matrix $\mathbf{T}$ naturally corresponds to a social network. The entry $T_{ij}$ describes the “trust” or “weight” that agent $i$ places on the beliefs of agent $j$ in forming his next-period beliefs.
Friendships at Westridge School

Under some fairly mild conditions, the belief of each individual $i$ eventually settles down to some limit

$$b_i(\infty) = \lim_{t \to \infty} b_i(t).$$
Now let us consider a sequence of societies, with agents $A_n$. We assume $|A_n| = n$. 
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Each society \( n \) has an associated vector of beliefs evolving over time: \( b^{(n)}(t) \).
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Each society $n$ has an associated vector of beliefs evolving over time: $b^{(n)}(t)$.

Assume beliefs in every society converge; let the vector of limiting beliefs in society $n$ be $b^{(n)}(\infty)$. 
Wisdom means that, as society grows large, limiting beliefs converge to the truth.
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**Definition**

The sequence \((T^{(n)})\) is *wise* if

\[
\lim_{n \to \infty} \max_{i \in A_n} |b_i^{(n)}(\infty) - \theta| = 0.
\]
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Denote by $T_{ij}(p)$ the $(i,j)$ entry of $T^p$.

Write

$$T_{i,B}(p) = \sum_{j \in B} T_{ij}(p).$$
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Prominent Groups

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**Definition**

The group $B$ is *prominent in $p$ steps* relative to $T$ if for each $i \notin B$, 

$$\pi_B(T; p) = \min_{i \notin B} T_i, B(p) > 0.$$
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The group $B$ is *prominent in $p$ steps* relative to $T$ if for each $i \notin B$, we have $T_{i,B}(p) > 0$.

Call $\pi_B(T; p) = \min_{i \notin B} T_{i,B}(p)$ the *$p$-step prominence* of $B$ relative to $T$. 
Example of a Prominent Group

The group in the dashed circle is prominent in 2 steps. Note that the rest of $T$ can be completed arbitrarily.
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Intuitively: \((B_n)\) is uniformly prominent with respect to \((T^{(n)})\) means:

- Each \(B_n\) is a prominent group with respect to \(T^{(n)}\).
- The prominence does not decay to 0.
Prominent Families: What We Are Ruling Out

$n = 10$

$n = 15$

$n = 20$
The family \((B_n)\) is *uniformly prominent* relative to \((T^{(n)})\)
Prominent Families: Formal Definition

Definition

The family \((B_n)\) is *uniformly prominent* relative to \((T^{(n)})\) if there exists a constant \(\mu > 0\) so that for each \(n\), there is a \(p\) so that

\[
\pi_{B_n}(T; p) \geq \mu.
\]
Proposition

If there is a finite, uniformly prominent family with respect to $(T^{(n)})$, then the sequence is not wise.
Intuition
Intuition

A Positive Result

$t = 0$

Small Prominent Families Prevent Wisdom

Intuition

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Naive Learning in Social Networks
Intuition

$t = 1$

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A network satisfies *minimal out-dispersion* if, for every finite family \((B_n)\) and every family \((C_n)\) with \(|C_n|/n \to 1\) we have \(T_{B_n,C_n} > r > 0\).

**Theorem**

If \((T^{(n)})\) satisfies balance and minimum out-dispersion, then it is wise.
Small prominent groups (media, pundits) are bad for information aggregation when agents are naive.

Balance and dispersion conditions can guarantee wisdom.
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- How many “good pollsters” do we need to add to ensure efficient learning, even if the initial structure is very bad?
- Interpolate between purely behavioral and purely rational learning.
- Nonhomogeneous updating (updating matrix changes).