EXPECTATIONS, NETWORKS, AND CONVENTIONS

Consider a situation where agents care about matching two targets—uncertainty about both:

- others’ actions;
- a “fundamentally” best action.

Conventions (in organizations, choice of language, speculative trading...): actions selected in equilibrium when coordination is important.

Question: How do conventions depend on differences in

(i) information (signals)
(ii) interpretation (priors)
(iii) coordination concerns (interaction)

beliefs and higher-order beliefs
networks

Contribution: Analyze effects of (i), (ii), (iii) together via reduction of all three to a network. Yields unification and new purely informational results.
**Model**

Agents
External state
i's fundamental
i's types
belief f'n.

\( i \in \mathbb{N} \)
\( \theta \in (0,1) \)
\( y^i : \Theta \rightarrow [-M,M] \)
\( t^i \in T^i \)
\( \pi^i : T^i \rightarrow \Delta(\Theta \times T^{-i}) \)
\( E^i \) i's expectation

strategy
\( a^i : T^i \rightarrow [-M,M] \)

\[ B R^i = \beta \sum_{j \neq i} \gamma^{ij} E^i a^j + (1-\beta) E^i y^i \]

**Ex post payoff**

\[ U^i = -\beta \sum_j \chi^{ij}(a^i - a^j)^2 \]

\[ -(1-\beta) (a^i - y^i(\theta))^2 \]
**Ex.** Net Game, Complete Info.

\[ u^i = -\beta \sum_j \gamma^{ij} (a^i - a^j)^2 - (1-\beta)(a^i - y^i)^2 \]

**Ex.** 2 agents, incomplete info

\[ u^i = -\beta (a^i - a^j)^2 - (1-\beta)(a^i - y^i(\theta))^2 \]

\[ \Theta \in \{G, B\} \]

\[ \rho^i \in \Delta(\Theta) \text{ i's prior} \]

\[ \pi^i \in \{g^i, b^i\} \text{ matches } \Theta \text{ w.p. } q^i \]

\[ \Pi^i \text{ computed via Bayes' rule} \]
**Model**

Agents

External state

i's fundamental belief f'n.

i's types

strategy

\[ y^i : T^i \rightarrow [-M, M] \]

\[ \pi^i : T^i \rightarrow \Delta(\Theta \times T^{-i}) \]

\[ E^i : i's\ expectation \]

\[ a^i : T^i \rightarrow [-M, M] \]

\[ BR^i = \beta \sum_{j \neq i} y^{ij} E^i a^j + (1-\beta) E^i y^i \]

- matching others' actions
- matching fundamental

\[ \text{Ex} \]

Net Game, Complete Info.

\[ u^i = \beta \sum_j y^{ij} (a^i - a^j)^2 + (1-\beta) (a^i - y^i)^2 \]

\[ y^1 \rightarrow y^2 \rightarrow y^3 \]

- 2 agents, incomplete info

\[ u^i = \beta (a^i - a^j)^2 + (1-\beta) (a^i - y(\Theta))^2 \]

\[ \Theta \in \{G, B\} \]

\[ \rho^i \in \Delta(\Theta) \text{ i's prior} \]

\[ t^i \in \{g^i, b^i\} \text{ matches } \Theta \text{ w.p. } q^i \]

\[ \pi^i \text{ computed via Bayes' rule} \]
**MODEL**

Agents

External state

i's fundamental belief f'n.

i's types

strategy

\[ \theta \in \Theta \]

\[ y^i : \Theta \rightarrow [-M, M] \]

\[ t^i \in T^i \]

\[ \Pi^i : T^i \rightarrow \Delta(\Theta \times T^{-i}) \]

\[ E^i \quad i's \ expectation \]

\[ a^i : T^i \rightarrow [-M, M] \]

\[ u^i = \beta (a^i - a^j)^2 + (1-\beta)(a^i - y^i(\theta))^2 \]

\[ \rho^i \in \Delta(\Theta) \quad i's \ prior \]

\[ t^i \in \{g^i, b^i\} \quad matches \theta \quad w.p. \ q^i \]

\[ \Pi^i \ \text{computed via Bayes' rule} \]

**FACT 1** The game has a unique \textit{rationalizable} strategy profile.

**Question:** How does play depend on (i) information; (ii) priors; (iii) network?

**Focus:** Conventions: play as $\beta \uparrow 1$.

**Example** 2 agents, incomplete info:

\[ BR^i = \beta \sum_{j \neq i} y^{ij} E^i a^j + (1-\beta) E^i y^i \]
**Key Idea:** Incomplete-info. aspect can be reduced to network aspect

**Key Device:** "Interaction structure"
- **Nodes:** \( S = \bigcup_i T^i \)
- **Edges:** \( B(t^i, t^j) = \gamma^{ij} \Pi^i(t^j | t^i) \)

**Fact 1:** The game has a unique rationalizable strategy profile.

**Question:** How does play depend on (i) information; (ii) priors; (iii) network?

**Focus:** Conventions: play as \( \beta \rightarrow 1 \).

**Fact 2:** The unique rationalizable action profile is given by

\[
 a = (1-\beta)(I - \beta B)^{-1} f
\]

**Example:** 2 agents, incomplete info.

\[
 -u^i = \beta (a^i - a^j)^2 + (1-\beta)(a^i - y(\theta))^2
\]

\( \theta \in \{G, B\} \)

\( \rho^i \in \Delta(\Theta) \) i's prior

\( t^i \in \{g^i, b^i\} \) matches \( \theta \) w.p. \( q^i \)

\( \Pi^i \) computed via Bayes' rule.
**KEY DEVICE:** “interaction structure”

\[ S = \bigcup_i T^i \quad B(t^i, t^j) = \gamma_{ij} \Pi^i(t^j | t^i) \]

Shin and Williamson (*GEB* 96) “How Much Common Belief is Necessary for a Convention?”

Morris (1997) “Interaction Games”


**Prop 1**

\[ c(\vec{y}; \vec{\pi}, \vec{\Gamma}) = \sum_{t^i \in S} p(t^i) f(t^i) \]

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( pB = p \)

I.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.

**Prop 0:** If \( B \) str. connected, then as \( \beta \uparrow 1 \),

\[ \forall i \ A^i(t^i) \rightarrow c(\vec{y}; \vec{\pi}, \vec{\Gamma}) : “the \ convention” \]
**Key Idea:** Incomplete-info. aspect can be reduced to network aspect → analyze how info. struct. matters.

**Key Device:** "interaction structure"

Nodes: $\mathcal{S} = \bigcup_i T^i$

Edges: $B(t^i, t^j) = \gamma^{ij} \Pi^i(t^j | t^i)$

**Applications**

1. Contagion of Optimism
2. (Pseudo) Common Prior
   - Influence $\propto$ net centrality
3. Tyranny of least-informed
KEY DEVICE: “interaction structure”

nodes: \[ S = \bigcup_i T^i \]
edges: \[ B(t^i, t^j) = \gamma_{ij} \pi^i(t^j | t^i) \]

CONTAGION OF OPTIMISM
Suppose each \( i \) is certain each counterparty has \( E^j y \geq E^i y + \delta \), unless \( E^i y \geq \bar{f} \); in that case, \( E^j y \geq E^i y \).
Then \( c(y; \pi, \Gamma) \geq \bar{f} \)
Reason: for \( t^i \) s.t. \( f(t^i) < \bar{f} \), the B process can only move upward.

PROP 1 \( c(y; \pi, \Gamma) = \sum_{t^i \in S} p(t^i) f(t^i) \)
where \( p \) is unique \( p \in \Delta(S) \) s.t. \( pB = p \)
I.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.
**KEY DEVICE:** "interaction structure"

**nodes:**

\[ S = \bigcup_i T^i \]

**edges:**

\[ B(t^i, t^j) = \gamma^{ij} \Pi^i(t^j | t^i) \]

---

**CONTAGION OF OPTIMISM**

Suppose each \( i \) is second-order optimistic (on avg)

\[ \frac{\sum_{j} \xi^{ij} E^i y_j}{\xi^i} \geq E^i y + \delta, \text{ unless } E^i y \geq \bar{f}; \text{ in that case, } \frac{\sum_{j} \xi^{ij} E^i y_j}{\xi^i} \geq E^i y - \epsilon. \]

Then

\[ c(y; \bar{\pi}, \Gamma) \geq \bar{f} / (1 + \epsilon/\delta) \]

Reason: for \( t^i \) s.t. \( f(t^i) < \bar{f} \), \( B \) process moves upward on average.

---

**PROP 1**

\[ c(y; \bar{\pi}, \Gamma) = \sum_{t^i \in S} p(t^i) f(t^i) \]

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( pB = p \)

i.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.
Harrison and Kreps (QJE 1978) “Speculative Investor Behavior...”
Izmalkov and Yildiz (AEJ:Micro 2010) “Investor Sentiments”
Han and Kyle (MS 2017) “Speculative Equilibrium with Differences in Higher-Order Beliefs”

CONTAGION OF OPTIMISM

Suppose each $i$ is second-order optimistic (on avg) $\sum_{j} E^i_j y \geq E^i y + \delta$, unless $E^i y \geq \bar{f}$; in that case, $E^i y \geq E^i y - \epsilon$.

Then $c(y; \pi, \Gamma) \geq \bar{f} / (1 + \epsilon/\delta)$.

Reason: for $t^i$ s.t. $f(t^i) < \bar{f}$, B process moves upward on average.

\[
\text{Prop 1: } c(y; \pi, \Gamma) = \sum_{t^i \in S} p(t^i) f(t^i)
\]

where $p$ is unique $p \in \Delta(S)$ s.t. $pB = p$

i.e. $p$ is the stationary distribution of $B$, viewed as a Markov chain.
**Key Device:** “interaction structure”

\[ S = \bigcup_i T^i \quad B(t^i, t^j) = \gamma^j \Pi^i(t^j | t^i) \]

**Contagion of Optimism**

Suppose each \( i \) is second-order optimistic

\[ \sum_i \gamma^i \Pi^i y \geq E^i y + \delta, \]

unless \( E^i y \geq \bar{f} \)

then \( \gamma^i \geq E^i y - \varepsilon. \)

Then \( c(y, \bar{\pi}, \bar{\pi}) \geq f(1 + \varepsilon/\delta) \)

Reason: for \( t^i \) s.t. \( f(t^i) < \bar{f} \), the \( B \) process moves upward on average

\[ \text{Prop 1} \quad c(y, \bar{\pi}, \bar{\pi}) = \sum_{t^i \in S} p(t^i) f(t^i) \]

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( pB = p \)

i.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.

**Proof:** Take MC \( W_0, W_1, \ldots \), with ergodic dist \( p \). Suppose \( \exists \delta, \varepsilon \) s.t.

\[ f(s) < \bar{f} \Rightarrow E_{W_0} [f(W_1)] \geq f(s) + \delta \]

\[ f(s) \geq \bar{f} \Rightarrow E_{W_0} [f(W_1)] \geq f(s) - \varepsilon \]

Then \( p(s : f(s) \geq \bar{f}) \geq \frac{1}{1 + \varepsilon/\delta} \)

Follows from

\[ E_{W_0 \sim p} [W_1 - W_0] = 0 \]
Higher Order Average Expectations

\[ x_{t^i}^i(1) = E^i[y^i|t^i] \]

1st-order expectation of \( y^i \) given i's info

\[ x_{t^i}^i(n+1) = \sum_j y_{ij} E^i[x_{t^i}^i(n)|t^i] \]

(n+1)th-order avg. expectation an average of \( n \)th-order exp. given i's info

Relation to Game

\[ [B^nf](t^i) = x_{t^i}(n+1) \]

\[ a_{eqm} = (1-\beta)(I-\beta B)^{-1}f = (1-\beta)\sum_{n=0}^{\infty} \beta^n B^n f \]
Higher-Order Average Expectations

\[ x_{t_i}^i(1) = \mathbb{E}^i[y_i | t_i] \]

1\textsuperscript{st}-order expectation of \( y_i \) given \( i \)'s info

\[ x_{t_i}^i(n+1) = \sum_j y_{ij} \mathbb{E}^i[x_{t_j}^j(n) | t_i] \]

\( (n+1)\textsuperscript{th} \)-order avg. expectation an average of \( n\textsuperscript{th} \)-order exp. given \( i \)'s info

Relation to Game

\[ \alpha_{eqm} = (1 - \beta) (I - \beta B)^{-1} f = (1 - \beta) \sum_{n=0}^{\infty} \beta^n B^n f \]

Samet (JET 98) “Iterated Expectation and Common Priors”

\[ a_{eqm}(t_i) = (1 - \beta) \sum_{n=0}^{\infty} \beta^n x_{t_i}^i(n+1) \]

Our companion paper: “Higher-Order Expectations”
**Common Priors & Influence**

**Prop 1**
\[ c(\bar{y}; \bar{\pi}, \bar{\pi}) = \sum_{t^i \in S} p(t^i) f(t^i) \]

where \( p \) is unique \( \pi \in \Delta(S) \) s.t. \( \pi B = p \)

I.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.

**Def** \( e(\Gamma) \) is defined as unique \( e \in \Delta(N) \) s.t. \( e \Gamma = e \).

**Prop 2**
\[ c = \sum_i e_i E y_i \]

where \( E y_i = \sum_{t^i \in T^i} \mu(t^i) E^i [y^i | t^i] \)

**Lemma** \( \forall i \)
\[ \sum_i p(t^i) = e_i \]
**COMMON PRIORS & INFLUENCE**

**Def** common priors over signals (CPs) $\pi^i$ all compatible w/ a $\hat{\pi} \in \Delta(T)$

$$\exists \text{ priors } (\mu^i)_{i \in N} \text{ s.t. } \mu^i(t^i) \pi^i(t^j | t^i) = \mu^i(t^j) \Pi^j(t^i | t^j)$$

**Def** $e(\Gamma)$ is defined as unique $e \in \Delta(\mathcal{N})$ s.t. $e \Gamma = e$.

**Prop 2** common prior $\Rightarrow C = \sum_i e^i E^i y^i$

where $E^i y^i = \sum_{t^i \in T^i} \mu^i(t^i) E^i [y^i | t^i]$

**Prop 1** $C(\bar{y}; \bar{\pi}, \bar{\Gamma}) = \sum_{t^i \in S} p(t^i) f(t^i)$

where $p$ is unique $p \in \Delta(S)$ s.t. $p B = p$

i.e. $p$ is the stationary distribution of $B$, viewed as a Markov chain.

**Lemma** $\forall i$

$$\sum_i p(t^i) = e^i$$
**Common Priors & Influence**

**Def.** Common priors over signals (CPSs)

\[ \pi^i \text{ all compatible } w/ \text{ a } \hat{\pi} \in \Delta(T) \]

\[ \exists \text{ priors } (\mu^i)_{i \in \mathbb{N}} \text{ s.t. } \mu^i(t') \pi^i(t'|t^i) = \mu^i(t') \pi^j(t'|t^j) \]

**Def.** \( e(\Gamma) \) is defined as unique \( e \in \Delta(N) \) s.t. \( e \Gamma = e \).

**Prop 1**

\[ c(y_j; \hat{\pi}, \bar{\pi}) = \sum_{t^i \in S} p(t^i) f(t^i) \]

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( pB = p \)

i.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.

---


Calvó-Armengol, de Marti, and Prat (TE 2015) “Communication and Influence”

Bergemann, Heumann, Morris (2017) "Information and Interaction“

Myatt and Wallace (2017), “Information Acquisition and Use by Networked Players”
**Tyranny of Least-Informed**

**Prop 3** Suppose $q^i \leq 1 - \delta$ at least $\delta$-noisy for all $i \neq 1$ $q^i \geq 1 - \epsilon$ at most $\epsilon$-noisy.

Then

$$|c(y; \pi) - \mathbb{E}_\mathbf{\hat{p}}^1[y]| \leq K \cdot \frac{\epsilon}{\delta}$$

**Prop 1** $c(y; \pi, \mathbf{\hat{p}}) = \sum_{t' \in S} p(t') f(t')$

where $p$ is unique $p \in \Delta(\mathcal{S})$ s.t. $p B = p$

i.e. $p$ is the stationary distribution of $B$, viewed as a Markov chain.

**Ex** 2 agents, incomplete info

$-u^i = \beta (a^i - a^j)^2 + (1 - \beta) (a^i - y(\theta))^2$

$\theta \in \{\theta_1, \ldots, \theta_K\}$

$p^i \in \Delta(\Theta)$ $i$'s prior

$t^i \in \{t^i_1, \ldots, t^i_K\}$ matches $\theta$ w.p. $q^i$

Otherwise full support noise.
Prop 3 Suppose $q^i \leq 1 - \delta$ at least $\delta$-noisy for all $i \neq 1$ $q^i \geq 1 - \varepsilon$ at most $\varepsilon$-noisy

Then

$$|c(y; \hat{\pi}) - \mathbb{E}^\hat{\pi}[y]| \leq K \cdot \frac{\varepsilon}{\delta}$$

Proof Idea

0. Define artificial $\hat{\pi}$:
   - each $i \neq 1$ knows $\Theta$
   - 1's info. unchanged

1. $c(y; \hat{\pi}) = \mathbb{E}^\hat{\pi}[y]$ Reason: $\hat{\pi}$ satisfies CPS with 1's prior.

2. $p(B_{\hat{\pi}}) \approx p(B_{\hat{\pi}})$
   Reason: if $\|B_{\hat{\pi}} - B_{\hat{\pi}}\| \times [\max. \text{ mean 1st-passage time in } B_{\hat{\pi}}]$ is small then $\approx$ holds.

Cho and Meyer (00) “Markov chain sensitivity measured by mean first passage times”
CONCLUSION

Interaction structure captures (interim) beliefs and network simultaneously: a method for studying how behavior depends on

(i) information (signals)  (ii) interpretation (priors)  (iii) coordination concerns (network)

General characterization of conventions in terms of eigenvector centrality in interaction structure. Reduction to a complete-information network game.

Illustrate with three applications.

Contagion of optimism – small local bias (in common direction) leads to extreme conventions.

Under common prior over signals, agents’ prior expectations matter in proportion to their centrality in the network $\Gamma$ only.

Under common interpretation of signals and precise private information, get tyranny of the least-informed.