

Social Choice Notes

May 21, 2005

1 May's Theorem

1.1 MWG's proof

May's Theorem argues that majority rule is the only social welfare functional that satisfies a trio of desirable properties in choosing between two social alternatives.

Those three properties are:

- Symmetry/anonymity: all that matters is what preferences are, not whose preferences they are (formally, scrambling the list of individual preferences will not change the social choice);
- Neutrality: the social choice rule must not favor one alternative or the other (formally, reversing everyone's preferences reverses the social choice); and
- Positive responsiveness: if any individual changes his preferences, the social choice must not move in the opposite direction

MWG's proof of May's Theorem¹ proceeds as follows. First, they show that any social welfare functional that satisfies symmetry and neutrality produces a tie when there are equal numbers of people strictly preferring the two alternatives. They show this in a cute but somewhat unintuitive way: imagine a particular reordering of preferences in this case (where there are equal numbers of people strictly preferring each of the two alternatives) where each person who favors alternative A changes place with someone who favors alternative B. Since there are equal numbers of people who favor A and B, this amounts to everyone with a strict preference switching that preference. By symmetry, this reordering cannot change the social preference. But by neutrality, switching every individual's preference must result in the opposite social choice. The only social choice which can both be reversed and not be changed is indifference – a tie.

So far, the proof has not narrowed down the domain of admissible social welfare functionals by much. For example, a social welfare functional that produces social indifference regardless of individual preferences satisfies both symmetry and neutrality. All we've done so far is make one parsimonious observation about what the combination of these two properties requires of a social welfare functional.

The next part of the proof simply points out that by adding positive responsiveness we ensure that the social welfare functional chooses the alternative favored by the most people (ie majority rule). Positive responsiveness means that from our 50/50 tie above, if one person moves in one direction or the other, the social choice also moves in that direction. These three properties thus describe majority rule.

¹Chris Avery's notes on social choice makes the same argument.

1.2 Discussion

This proof is interesting and parsimonious. The way to make sure that it makes sense is to think of social choice functionals that meet subsets of the three May identified. At first I thought that positive responsiveness and neutrality alone might identify majority rule: neutrality assures that the decision rule is centered around a 50/50 split, with a potentially large band of indifference in the middle, and positive responsiveness says that the band of indifference can be only one vote. But implicitly I am assuming a voting rule here and ignoring other possibilities, such as the lexicographic rule that Avery considers in his notes: if person 1 has a strict preference, that is the social preference; if not, the choice goes to person 2, and so on. This rule satisfies neutrality, but has nothing to do with a voting rule.

Here is the simplest way I can make the argument:

- Symmetry guarantees a one-person, one-vote decision rule;
- Neutrality guarantees that the voting rule would be centered around zero, with a possibly large band of indifference in the center; and
- Positive responsiveness guarantees that the band of indifference be only one person (a tie) and that the social choice reflects the majority rather than the minority opinion.

Incidentally, Ben (and Jerry Green?) also mentioned the Pareto property in laying out May's Theorem, but I think that positive responsiveness and neutrality together ensure that unanimity in favor of A results in the social choice of A.