Math E-15 Homework 7

1. Consider the family of curves $f(x) = x^ne^{-7x}$ where $n$ is a positive even integer.

   (a) Find the roots of $f$. (That is, solve $f(x) = 0$.)

   (b) Find the $x$- AND $y$-coordinate(s) of all critical points and classify each as a local maximum, local minimum, or neither.

   (c) Sketch $f$, labeling all the points you found above. Note that this question does not ask you to find the inflection points (though doing so is certainly good practice).

2. You wish to build an outdoor rectangular enclosure for your pets. One side of the rectangle will be one of the outer walls of your house. If you have 80 feet of fencing, what are the dimensions that will enclose the maximum possible area? Be sure to show how you know you have found the absolute maximum.

   Remember that you do NOT need to use fence for the side that is already there (the wall of your house).

3. As shown below, a newly drilled oil well is 7 kilometers offshore and 10 kilometers east of an oil refinery that is already in place on the shoreline. If connecting pipe costs $5 million per kilometer in the water and $3 million per kilometer on the shore, what path will minimize the total cost of connecting the new well to the refinery? Be sure to show how you know you have found the absolute minimum.

   Please include a sketch that shows the amount of pipe in the water and the amount on the shoreline.